

Admissible Coarse-to-Fine Continuation of Reacting Hypersonic Flow on Native Hexahedral Grids

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A developed reacting-air solution is transferred to an unchanged, 41,962,496-hexahedron domain through an admissibility-checked continuation method. Seven species, two thermal energies, and shear-stress-transport turbulence are coupled through thermodynamic, acoustic, charged-transport, and nonlinear-update equations. A native trilinear inverse map and density-weighted turbulence interpolation locate 41,806,828 of 42,396,569 receiver nodes. The remaining 589,741 nodes retain an independently accepted fine-grid state. Native wall, thermodynamic, and turbulence recovery admits every receiver node, with maximum thermal and modal energy roundtrip errors of 2.08×10^{-14} and 1.44×10^{-14} . A bounded-memory backend preserves phase ordering across 64 ownership partitions, combines cached topology with ordered graphics-processor replay in software binary64 arithmetic, and checks processor-side turbulence corrections using recomputed global and equation-wise double-precision residuals. Matched complete-update measurements yield bitwise-identical state and numerical carry; timing comparisons retain their measured workload conditions. Ten transferred fine-grid updates pass native admissibility and linear-direction checks in 568–631 seconds per update. Thermal, species, residual, and matched-coordinate compression diagnostics quantify the continuation; decreasing energy norms and increasing turbulence norms identify ongoing relaxation. The contribution integrates complete-state transfer, quantified geometric disagreement, native recovery, and reproducible full-grid execution for continued convergence and physical assessment.

Nomenclature

E	= total specific energy, J/kg
e_v	= vibration–electronic specific energy, J/kg
h_f	= species formation enthalpy, J/kg
k	= turbulent kinetic energy, m^2/s^2
M, M_n	= total and normal Mach number
N_c, N_p	= numbers of cells and points
p	= pressure, Pa
R_s	= species gas constant, J/(kg K)
r	= algebraic residual vector
T, T_v	= translational and modal temperature, K
u	= velocity vector, m/s
V_i	= nodal control volume, m^3
Y_s	= species mass fraction
α_f	= face antidiffusive-flux coefficient
δ_{10-90}	= sampled pressure-rise width, m
μ, μ_t	= molecular and eddy viscosity, Pa s
ρ, ρ_s	= mixture and species density, kg/m^3
τ	= numerical pseudo-time
ω	= specific dissipation rate, s^{-1}

I. Introduction

Reacting hypersonic flow couples sharply varying compression, wall transport, chemical production, internal-energy relaxation, and turbulent stresses. On a finely resolved three-dimensional mesh, an economical calculation must preserve these interactions while reducing both the cost of an update and the number of updates needed to develop useful flow structure. This study connects equation-level extensions of SU2-NEMO to a verified coarse-to-fine initialization and a bounded-memory native execution path on a full hexahedral domain assembled at the supplied grid’s original resolution.

HIFiRE provides an aeropropulsion context with closely coupled external and internal flow paths [28]. Bisek’s HIFiRE6 simulations identify sensitivity of mass capture, separation, and thermal response to angle of attack and downstream confinement [2]; Mileski develops a surrogate inlet for investigation of the associated

shock/boundary-layer interaction [3]. Integrated propulsion models address a complementary systems question through efficient inlet, combustor, and nozzle representations [1]. Spatially resolved reacting calculations expose the local processes required to assess the assumptions in such representations. The present supplied geometry is therefore accompanied by explicit operating-condition, coordinate, boundary, and experimental-applicability statements.

SU2-NEMO supplies a documented multispecies, two-temperature framework [4, 5], while SST supplies a widely studied eddy-viscosity closure [6, 7]. The local implementation couples these components through a consistent total-energy convention, mode-specific turbulent transport, charge-preserving species fluxes, thermodynamic and acoustic derivatives, and segregated state-admissibility controls. Each inherited model and each local modification is identified. Contemporary hypersonic SST comparisons show why the precise stress convention, near-wall treatment, numerical flux, and experimental condition matter when interpreting separation and heating [20, 25, 26, 9].

Coarse-to-fine initialization and multilevel acceleration have an established numerical history [13]. Transfer between nonmatching meshes introduces additional requirements: geometric location, admissibility, boundary compatibility, and the choice between nodal interpolation and integral conservation [14, 15, 16]. The contribution developed here is the implementation and source-bound verification of these operations for the coupled AIR-7/two-temperature/SST state, including modal energy and the turbulent energy already embedded in total energy. A measured containment audit identifies the exact receiver points assigned to interpolation or an explicitly retained fallback. Native fine-wall recovery then preserves the intended internal energies while enforcing the receiver’s no-slip and SST wall conditions.

The supplied 20,981,248-cell half-domain is reflected and joined at $y = 0$, producing 41,962,496 hexahedra and 42,396,569 unique nodes. This joined source-resolution receiver remains unchanged during solution transfer, with no additional donor wake region. The full three-dimensional state is advanced without an imposed symmetry boundary condition at the join. A phase-synchronous ownership scheme, reusable geometry tiles, ordered buffered state writes, thread-private native thermochemistry, and qualified GPU accumulation stages execute the full discrete update within a shared 48 GiB memory architecture. Same-input experiments measure throughput while testing complete state and numerical-history equivalence.

Table 1 Specified computational conditions

Quantity	Value
Freestream Mach number	6.0
Pressure, Pa	1897.27354577
$T_\infty = T_{v,\infty}$, K	223.47222222
Angle of attack, deg	0
Y_{N_2}	0.76708248854
Y_{O_2}	0.23291751146
Other freestream mass fractions	0
Turbulence intensity	0.001
Freestream μ_t/μ	1.0
Turbulent Prandtl number	0.9
Turbulent Schmidt number	0.7

The native thermodynamic equations are retained throughout this execution path.

The analysis joins this execution evidence to high-resolution field assessment. Three-dimensional pressure surfaces locate compression geometry; native pressure-normal profiles test plateau quality, normal Mach crossing, flux changes, and numerical rise width. Dense velocity integral curves and conservative velocity vectors reveal turning and separation within explicitly retained native cells. Thermochemical views state physical values and trace abundance. All fourteen phase residual norms, linear-direction checks, and update restrictions remain separate measurements. The retained donor study and the ten-update fine-grid continuation are presented together, with their different domains, freestream pressures, initialization histories, and stages of numerical development identified in each comparison.

II. Physical Model and Donor Computational Case

A. Study Definitions and Conditions

The application is a three-dimensional, unpowered reacting-air calculation on a supplied HIFIRE6-labeled GridPro geometry. Results are closed, developing numerical iterates, with iterative convergence, grid and domain sensitivity, and experimental agreement treated as separate acceptance objectives. The principal calculations use steady local pseudo-time: the iteration index records numerical development, and elapsed execution time records computational cost. Physical times are assigned only to the separately identified frozen-air forecast experiment.

The imposed freestream and transport parameters appear in Table 1. Neutral air is specified by nitrogen and oxygen mole fractions of 0.79 and 0.21, converted to the listed mass fractions using the selected molecular masses. Fully turbulent flow uses SST V2003m with prescribed inlet intensity and eddy-to-molecular viscosity ratio. Walls are stationary, smooth, no-slip, adiabatic, and noncatalytic. The computational geometry and operating point define the present case; correspondence to a particular flight or facility experiment requires a separately matched configuration and wall/transition specification.

The coarse supplier GridPro half-domain contains 871,250 hexahedra and 909,388 points; reflection and seam joining yield the 1,742,500-hexahedron, 1,791,596-point baseline used in the present SU2 runs. A separate fine supplier half-domain contains 20,981,248 hexahedra and 21,315,388 points, corresponding to 41,962,496 cells after reflection. The retained donor results compare the coarse baseline with its directional refinement. The new continuation uses the separate original fine supplier grid without the added outer or wake domain. SU2 operates on the supplied face-connectivity representation; GridPro-derived hexahedral topology is preserved through subsequent processing. The coarse baseline outlet lies at $x = 4.305532$ m and the outer cylindrical radius is 0.774047 m. The aftmost wall or appendage reaches $x = 2.982584$ m. These measured coordinates establish the available downstream distance independently of a nominal vehicle-length convention.

III. Implementation of the Coupled Equations

A. Equation Set and Implementation Scope

The implementation joins NEMO's reacting two-temperature equations to the native modified 2003 shear-stress-transport (SST) closure [4, 6, 7]. The AIR-7 ordering is e^- , N_2 , O_2 , NO , N , O , and NO^+ . Seven species densities, three momentum components, total energy density, and modal energy density form the twelve-component flow block. The SST state stores specific k and ω and uses conservative density-weighted increments. The changes comprise thermodynamic consistency corrections, coupled turbulent transport, charged-pair constraints, and a segregated solution path with state-admissibility controls.

Let $q_E = \rho E$, $q_v = \rho e_v$, $m = \rho u$, and $\rho = \sum_s \rho_s$. With $F_s = -J_s$ denoting the positive viscous-flux convention, the implemented balances are

$$\begin{aligned}\partial_t \rho_s + \nabla \cdot (\rho_s u) &= \nabla \cdot F_s + \dot{\omega}_s, \\ \partial_t m + \nabla \cdot (m \otimes u + pI) &= \nabla \cdot \tau, \\ \partial_t q_E + \nabla \cdot [(q_E + p)u] &= \nabla \cdot F_E, \\ \partial_t q_v + \nabla \cdot (q_v u) &= \nabla \cdot F_v + S_v.\end{aligned}\quad (1)$$

The calculation seeks a steady solution with local pseudo-time updates. Formation energies are included in q_E , so chemical production enters the species and modal balances without an additional total-energy heat-release source. The modeled flow contains air reactions; its species set and boundary conditions contain no fuel or injector.

B. Caloric State, Reference Energies, and Turbulent Energy

For molar masses M_s in kg/kmol, define $R_s = \mathcal{R}/M_s$, with $\mathcal{R} = 1000R_u$. Heavy species use translational-rotational temperature T ; electron translation and heavy-particle vibrational and electronic energies use T_v . The heavy-particle caloric offset and energy are

$$\begin{aligned}b_s &= h_{f,s}^0 - (c_{v, \text{tr}, s} + R_s)T_s^0, \\ e_{\text{tr}, s}(T) &= c_{v, \text{tr}, s}T + b_s, \quad s \neq e,\end{aligned}\quad (2)$$

with $b_e = c_{v, \text{tr}, e} = e_{\text{tr}, e} = 0$. Here $h_{f,s}^0$ and T_s^0 are the species formation enthalpy and reference temperature. The inherited species function $e_{v,s}(T_v)$ includes electron translational and formation energy for $s = e$. The patch evaluates mixture energy, species enthalpy, temperature recovery, and their derivatives using this same decomposition:

$$\begin{aligned}q_v &= \sum_s \rho_s e_{v,s}(T_v), \\ q_E &= \sum_s \rho_s e_{\text{tr}, s}(T) + q_v + \frac{|m|^2}{2\rho} + \rho k.\end{aligned}\quad (3)$$

The translational temperature therefore satisfies

$$T = \frac{q_E - q_v - |m|^2/(2\rho) - \rho k - \sum_s \rho_s b_s}{C_{\text{tr}}}, \quad (4)$$

where $C_{\text{tr}} = \sum_{s \neq e} \rho_s c_{v, \text{tr}, s}$ is a volumetric heat capacity. The implemented mixture gas constant is $R_{\text{mix}} = \sum_s Y_s R_s$, replacing the reciprocal of a mass-fraction-weighted molar mass. Heavy-species sums include the final AIR-7 ion and exclude the electron. The ion molar mass is corrected to $M_{\text{NO}^+} = M_N + M_O - M_e$, retaining the mass of the separately represented electron.

The two-temperature pressure law is inherited from NEMO:

$$p = AT + BT_v, \quad A = \sum_{s \neq e} \rho_s R_s, \quad B = \rho_e R_e. \quad (5)$$

Specific total enthalpy is $H = (q_E + p)/\rho$. At MUSCL endpoints, reconstructed species densities, temperatures, and velocity are passed through the same gas closure; $H = e + |u|^2/2 + k + p/\rho$ and $q_E = \rho H - p$ retain k exactly once. This changes the thermodynamic inputs to the inherited AUSM-family flux, while retaining its splitting formula. Viscous and reconstruction gradients occupy separate storage: diffusion uses ∇Y_s , whereas MUSCL reconstructs ρ_s .

C. Segregated Thermodynamic Derivatives and Acoustic Speed

For the flow Jacobian, k is fixed while each species density contributes to ρ . Let $C_v = \sum_s \rho_s c_{v,v,s}$, with $c_{v,v,s} = de_{v,s}/dT_v$. The corrected derivatives are

$$\begin{aligned} T_{,\rho_s} &= \frac{|u|^2/2 - k - e_{tr,s}}{C_{tr}}, & T_{,m_j} &= -\frac{u_j}{C_{tr}}, \\ T_{,q_E} &= C_{tr}^{-1}, & T_{,q_v} &= -C_{tr}^{-1}, \\ (T_v)_{,\rho_s} &= -\frac{e_{v,s}}{C_v}, & (T_v)_{,m_j} &= 0, \\ (T_v)_{,q_E} &= 0, & (T_v)_{,q_v} &= C_v^{-1}. \end{aligned} \quad (6)$$

The modal derivatives retain the native caloric relation. Equation (6) uses $C_v > 0$, as in this AIR-7 state; the conservative-recovery branch assigns zero modal derivatives when modal heat capacity is zero. The corrected pressure derivative includes both thermal chains and every species:

$$\begin{aligned} p_{,U_j} &= AT_{,U_j} + B(T_v)_{,U_j} + d_j, \\ d_j &= \begin{cases} R_s T, & U_j = \rho_s, s \neq e, \\ R_e T_v, & U_j = \rho_e, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (7)$$

In code, the gas derivative is evaluated first and the variable and reconstruction layers subtract $k p_{,q_E}$ and $k T_{,q_E}$ from each species-density column. This is the chain rule associated with removing ρk from thermodynamic energy; it changes no modal-energy derivative.

Writing $p_h = AT$ and $p_e = BT_v$, the frozen-composition sound speed used by the flux and timestep becomes

$$a_f^2 = \frac{p_h + p_e}{\rho} + \frac{p_h A}{\rho C_{tr}} + \frac{p_e B}{\rho C_v}. \quad (8)$$

The electron term is evaluated when $B > 0$, and the corresponding effective γ is $a_f^2 \rho / p$. This expression is consistent with electron compression work in the modal equation. It replaces application of the heavy-particle heat-capacity ratio to the complete pressure.

D. Turbulent Heat and Species Fluxes

The closure uses $\mu_{\text{eff}} = \mu + \mu_t$, $\text{Pr}_t = 0.9$, and $\text{Sc}_t = 0.7$. Molecular coefficients retain their implemented Gupta–Yos lineage [12]. A corrected composition index in the molecular conductivity mixture sum uses the collision partner's density ρ_j rather than ρ_i . The additional turbulent coefficients are

$$\begin{aligned} D_t &= \frac{\mu_t}{\rho \text{Sc}_t}, \\ c_{p,tr} &= \sum_{s \neq e} Y_s (c_{v,tr,s} + R_s), \\ c_{p,v} &= \sum_s Y_s c_{v,v,s} + Y_e R_e, \\ \kappa_{tr}^* &= \kappa_{tr} + \mu_t c_{p,tr} / \text{Pr}, \\ \kappa_v^* &= \kappa_v + \mu_t c_{p,v} / \text{Pr}. \end{aligned} \quad (9)$$

The electron pressure contribution belongs to $c_{p,v}$. The mixture-corrected preliminary species flux is

$$\begin{aligned} G_s &= \rho (D_s + D_t) \nabla Y_s, \\ \tilde{F}_s &= G_s - Y_s \sum_r G_r. \end{aligned} \quad (10)$$

For neutral mixtures this sums to zero mass flux. AIR-7 applies a linear charged-pair projection. With $i = \text{NO}^+$ and $P = \tilde{F}_e + \tilde{F}_i$,

$$F_e = \frac{M_e P}{M_e + M_i}, \quad F_i = \frac{M_i P}{M_e + M_i}. \quad (11)$$

Neutral fluxes retain their preliminary values. Thus $\sum_s F_s = 0$ and $F_e/M_e = F_i/M_i$. This is the implemented quasineutral, zero-current mixture-Fick approximation. It resolves neither an electric potential nor multicomponent Stefan–Maxwell transport. The discrete pair-mass update described below preserves the same constraint through residual assembly and state storage.

Define species temperatures $T_s = T$ for heavy species and $T_e = T_v$. The corrected species and modal enthalpies are

$$\begin{aligned} h_s &= e_{tr,s} + e_{v,s} + R_s T_s, \\ h_{v,s} &= e_{v,s} + \delta_{se} R_e T_v. \end{aligned} \quad (12)$$

The implemented diffusive energy fluxes are

$$\begin{aligned} F_E &= \tau \cdot u + \sum_s h_s F_s + \kappa_{tr}^* \nabla T + \kappa_v^* \nabla T_v, \\ F_v &= \sum_s h_{v,s} F_s + \kappa_v^* \nabla T_v. \end{aligned} \quad (13)$$

The modal contribution appears in the total-energy equation because modal energy is part of E . The heavy and modal heat capacities avoid counting electron pressure twice. Following the selected SST V2003m stress convention, $\tau = 2(\mu + \mu_t)[S - (\nabla \cdot u)I/3]$ contains no isotropic ρk term. The retained total-energy approximation omits the modeled k -diffusion contribution $(\mu + \sigma_k \mu_t) \nabla k$; this convention is separate from the stress convention.

E. Modal Source and Inherited Reversible Chemistry

The modal source combines heavy-particle relaxation, species production, and electron compression work:

$$\begin{aligned} S_v &= \Omega_{VT} + \sum_s \dot{\omega}_s e_{v,s} - p_e \nabla \cdot u, \\ \Omega_{VT} &= \sum_{s \neq e} \frac{\rho_s}{\tau_s} [e_{\text{vib},s}(T) - e_{\text{vib},s}(T_v)]. \end{aligned} \quad (14)$$

The patch includes every heavy species in the inherited Millikan–White/Park relaxation sums and excludes the electron from those heavy-particle sums. The relaxation energies are vibration-only; the chemical modal-energy contribution uses the full $e_{v,s}$. A separate electron–heavy collisional relaxation model is outside this closure. Electron pressure work is added to the modal balance only, outside the optional native relaxation-source limiter; that limiter is disabled in the qualified configuration.

For the local implicit approximation, velocity divergence is fixed, giving

$$\frac{\partial S_{v,e}}{\partial U_j} = -R_e (\nabla \cdot u) [\delta_{j,\rho_e} T_v + \rho_e (T_v)_{,U_j}]. \quad (15)$$

Neighbor-gradient derivatives are omitted. The retained VT Jacobian freezes relaxation times and uses the native modal-capacity approximation. The qualified mean-flow integration is explicit, so these approximate source blocks do not constitute a fully coupled Newton linearization.

The reaction table, Arrhenius coefficients, equilibrium-constant table, and forward/backward mass-action evaluation are inherited. For forward and backward controlling temperatures $T_r = T^a T_v^{b_r}$, the inherited regularization is

$$\hat{T}_r = \frac{T_r + 800 + \sqrt{(T_r - 800)^2 + 80^2}}{2}, \quad (16)$$

with temperatures in kelvin. Rates use $k_f = A_r \hat{T}_f^{\eta_r} \exp(-\theta_r / \hat{T}_f)$ and $k_b = k_f(\hat{T}_b) / K_{eq}(\hat{T}_b)$. Net species production is the molar-mass-weighted stoichiometric sum of forward minus backward reaction progress. The patch preserves this reversibility and regularization. It changes the ion mass and thermodynamic consistency around that mechanism. Production is evaluated from the mean species state and temperatures; no additional turbulence–chemistry probability-density or mixing-time model is supplied.

F. Discrete Gradient and Jacobian Consistency

Both Green–Gauss and least-squares viscous paths now construct mass-fraction gradients. Their flux routines consume these gradients directly, removing a second, inconsistent conversion from species-density to mass-fraction gradients. Reconstruction uses independent species-density gradients even when the selected gradient operators are identical. The uncorrected viscous Jacobian now uses endpoint distance instead of squared distance, and the corrected viscous routine returns its left and right blocks separately; the former return supplied the right block twice.

The viscous Jacobian uses the same effective viscosity, turbulent diffusivity, two thermal conductivities, species enthalpies, and charged-pair projection as the residual. Every species-diffusion Jacobian column receives the linear projection in Eq. (11). Electron enthalpy temperature derivatives place R_e in the modal column. This is a frozen-coefficient thin-shear-layer approximation: coefficient, composition-capacity, eddy-viscosity, and reconstructed-gradient derivatives are omitted. These corrections improve the defined linearization without representing an exact derivative of every nonlinear operation.

G. SST Integration, Boundaries, and Restart Semantics

The implementation allocates the existing SST solver and numerics for NEMO, computes wall distance, provides the larger reacting primitive layout, and selects explicit Euler FLOW with implicit Euler SST. The SST V2003m production, destruction, blending, cross-diffusion, and eddy-viscosity formulas retain their upstream coefficients. After each FLOW update, mixture properties and SST eddy viscosity are refreshed. After the SST update, the gas state is refreshed again because changing k changes the energy available to T at fixed q_E . The largest native nonphysical-point count across those stages is retained.

Farfield transport receives the consistent turbulent state. Qualified walls are stationary, no-slip, noncatalytic, adiabatic heat-flux walls without wall functions. The added inlet adapter evaluates density and molecular viscosity from species densities and both temperatures before forming the inherited $k = 3|u|^2 I^2 / 2$ and $\omega = \rho k / [\mu(\mu_t / \mu)]$ prescriptions. The supported external-flow configuration selects farfield boundaries and rejects inlet/outlet configurations, so that adapter is not used as a boundary-validation claim. No new wall-density or wall-viscosity derivative model is introduced.

Restart loading checks the named species, momentum, total-energy, modal-energy, k , and ω layout before using the state, and retains FP64 solution storage. FLOW's integration scheme is established before property recovery. Shared restart thermochemistry is serialized, while point-local gas evaluation uses independent thermochemical objects. Filename handling uses dynamically sized strings. Output convergence fields and residual indices cover all fourteen equations, and limiter output identifies the actual reconstructed primitive variables.

H. State Recovery and Verification Boundaries

Modal temperature is obtained from the evaluated caloric residual $r = \sum_s \rho_s e_{v,s}(T_v) - q_v$ on the native 50–80,000 K bracket. A safeguarded Newton/bisection iteration accepts only when

$$|r| \leq 64\epsilon_{\text{mach}} \max\left(\left|\sum_s \rho_s e_{v,s}\right|, |q_v|, \nu_{\min}\right),$$

$$r = 0 \quad \text{or} \quad |r/C_v| \leq 10^{-10} \max(1, |T_v|), \quad (17)$$

where $\nu_{\min}$ is the smallest positive normal FP64 value. The iteration is bounded by 128 trials. Newton proposals must be finite and strictly within the current bracket; otherwise bisection is used. Failure produces an explicit rejected state instead of assigning $T_v = T$. The variable layer rejects temperature endpoints, negative species, invalid capacities, nonfinite quantities, and energy round-trip disagreement. In strict coupled operation it records and rejects the original state before native recovery can substitute an earlier state.

These acceptance tests complement the conservative face correction, common FLOW timestep restriction, post-SST thermal relaxation, and true-residual linear checks developed in the next section. They preserve the selected equation set while controlling the discrete update. Source-bound tests include 22,394 thermodynamic checks using the patched gas and variable classes, transport and positivity harnesses, rejected-state tests, and native replay through the prior thermal failure points. Kernel checks, exact-source-body checks, and full-grid replay establish distinct levels of implementation evidence; the mathematical assumptions above remain explicit in each level.

IV. Integrated Numerical Method

A. Finite-Volume Update and State Acceptance

The nodal finite-volume formulation uses AUSM+up2 convection. Liou [8] provides the AUSM+ family formulation, and Kitamura and Shima [9] provide the pressure-flux development cited by the selected up2 implementation. MUSCL reconstruction is enabled for flow and turbulence with Green–Gauss gradients and the Venkatakrishnan–Wang limiter. The limiter combines smooth limiting [10] with the solution-range regularization associated with Wang's modification [11].

The reacting flow advances explicitly, while SST uses an implicit flexible-GMRES solve with block-Jacobi preconditioning. The historical coarse-grid higher-order continuation used nominal CFL 0.001. The refined donor continuation used CFL 0.5, twenty allowed SST linear iterations, and unchanged 10^{-7} global and equation-specific acceptance thresholds. Native CFL adaptation is disabled. Local restrictions below reduce the admissible pseudo-step according to the actual species and thermal state; therefore nominal CFL alone does not determine the local advance.

B. Positive Viscous Species and Energy Correction

The added flux-correction algorithm acts on viscous species transport and its consistent total/modal energy fluxes. Convective MUSCL endpoint fallback is a distinct path. For an edge joining i and j , let n_{ij} be the native dual-face area vector and Δx_{ij} its primal edge. A positive finite metric is required:

$$g_{ij} = \frac{n_{ij} \cdot \Delta x_{ij}}{|\Delta x_{ij}|^2} > 0, \quad K_s = \rho_f (D_{s,f} + D_{t,f}) g_{ij}. \quad (18)$$

Endpoint-averaged density, eddy viscosity, and molecular diffusivities define the face coefficients. AIR-7 groups the charged-pair mass fraction $Y_c = Y_e + Y_i$, with conductance $K_c = (M_e K_e + M_i K_i) / (M_e + M_i)$, before applying the following six-component low-order construction:

$$S = \sum_s K_s (Y_{j,s} - Y_{i,s}),$$

$$L_s = (K_s - \min(S, 0)) Y_{j,s} - (K_s + \max(S, 0)) Y_{i,s}. \quad (19)$$

The pair flux is then split by the same molar-mass ratio as the transport closure. Summing the grouped fluxes gives zero. The complete low-order residual includes the existing convection, chemical and modal sources, boundary contributions, and truncation terms. One local step is restricted so every depleting species retains a ten-percent inventory margin. This qualifies the actual assembled low-order candidate, rather than a diffusion-only surrogate.

Let $H_{f,s}$ be the native high-order viscous flux and $A_{f,s} = H_{f,s} - L_{f,s}$. All adverse incident corrections are accumulated before allocating a nodal inventory budget. For the sign in which positive flux increases node i and depletes node j ,

$$P_{i,s} = \sum_{f \ni i} \max(-A_{f,s}^{(i)}, 0),$$

$$r_{i,s} = \min\left(1, \frac{0.9 \rho_{i,s}^{LO}}{(\Delta \tau_i / V_i) P_{i,s}}\right). \quad (20)$$

Zero outgoing correction gives $r_{i,s} = 1$; zero available density with positive outgoing correction gives zero. The face coefficient is the minimum relevant endpoint/species fraction. It is shared by all seven species and both energy channels:

$$F_f^{lim} = (1 - a_f) L_f + a_f H_f, \quad 0 \leq a_f \leq 1. \quad (21)$$

Both endpoint residuals receive equal and opposite contributions. Low- and high-order energy fluxes are formed from their corresponding species fluxes with the same species enthalpies and non-species heat/stress terms. Exact endpoint selection at $\alpha = 0$ or 1 avoids cancellation when one flux is much smaller than the other. This construction preserves the spatial face pairing; varying nodal pseudo-steps do not imply a globally conservative physical-time advance.

The electron and ion are advanced using one pair mass coordinate,

$$\begin{aligned}\rho_e^{n+1} &= (\rho_e^n + \rho_i^n) - \frac{\Delta\tau}{V}(R_e + R_i), \\ (\rho_e^{n+1}, \rho_i^{n+1}) &= \frac{\rho_e^{n+1}}{M_e + M_i}(M_e, M_i).\end{aligned}\quad (22)$$

The input charge defect must satisfy $|\rho_e/M_e - \rho_i/M_i| \leq 10^{-7} \max(\rho_e/M_e, \rho_i/M_i)$. Splitting the updated pair corrects that accepted preexisting representation defect while preserving its mass. No trace-species floor is added. The retained positive-transport harness records 237,808 assertions, 43 deliberate refusals, 2,000 random faces spanning 240 fraction decades, and 20,000 sequential pair updates.

C. Diffusive and Thermal Pseudo-Step Restrictions

The diffusive estimate uses the actual molecular/eddy closure, with specific capacities $c_{v,tr} = C_u/\rho$ and $c_{v,ve} = C_v/\rho$. The native stored modal-energy label ρE_{ve} used below denotes q_v . With P_c the charge projection and Y the mass-fraction column, its frozen-block bound is

$$\begin{aligned}v_b &= \frac{4(\mu + \mu_i)}{3\rho} + \max\left(\frac{\kappa_{tr}^*}{\rho c_{v,tr}}, \frac{\kappa_v^*}{\rho c_{v,ve}}, \right. \\ &\quad \left. \|P_c(I - Y1^T) \text{diag}(D_s + D_t)\|_1\right).\end{aligned}\quad (23)$$

This restriction measures the frozen diffusive blocks. Chemical stiffness and the full nonorthogonal operator require the subsequent assembled-state tests.

After viscous correction, the final flow residual R_f is fixed. The explicit trial is formed using the actual charged-pair update and a single local step shared by all twelve flow components:

$$U_f(\Delta\tau) = U_f^n - (\Delta\tau/V)R_f. \quad (24)$$

At fixed pre-SST k , the algorithm halves $\Delta\tau$ until the native caloric state has $50 < T < 80000$ K and

$$\sum_s \rho_s^* e_{ve,s}(50) < \rho E_{ve}^* < \sum_s \rho_s^* e_{ve,s}(80000). \quad (25)$$

It allows at most sixty-four halvings, rejects nonrepresentable positive steps and restricted candidates with no representable progress, and leaves the actual state and residual unchanged until acceptance. This bounds the candidate's two-temperature recovery without clipping its conservative components.

The post-SST restriction addresses a different coupling: increasing k at fixed flow total energy reduces the translational energy available in Eq. (4). Holding all flow components and the accepted conservative SST direction fixed, it evaluates

$$\begin{aligned}q^{raw}(\alpha) &= \frac{\rho q^{old} + \alpha \Delta(\rho q)}{\rho}, \\ q^*(\alpha) &= \Pi_{[q_{min}, q_{max}]} q^{raw}(\alpha), \quad q = (k, \omega).\end{aligned}\quad (26)$$

The density is the current FLOW density in both terms. Π is the existing native componentwise SST bound; the thermal guard adds no new bound values. Starting from the existing common relaxation factor, it halves α until the actual bounded k^* leaves the fixed flow state thermally admissible. The same sixty-four-halving and representable-progress safeguards apply. Direction collinearity holds before componentwise projection. Counts of thermal restriction, native bound activity, and zero relaxation are reported separately.

The meanflow kernel passes 48 retained checks, including the actual failed run033 state, for which a halfstep recovers $T = 307.7918$ K. The post-SST kernel passes 30 checks. Its run035 fixture uses the observed failing flow state and failing k , with a declared constructed old- k anchor and direction because the original direction was not logged. An independent ten-update native replay subsequently crosses both former failure points and passes complete saved-field verification. This distinguishes a reproduced unit-level acceptance mechanism from the later executed coupled trajectory.

D. Closed-State Acceptance

Strict coupled-state recovery rejects nonfinite/negative species, non-positive mixture density, invalid k , nonfinite or out-of-range temperatures, and invalid heat capacities, pressure, or sound speed. Caloric roundtrip errors must be below $10^{-8} \max(1, |\rho E_{gas}|)$ and $10^{-8} \max(1, |\rho E_{ve}|)$, respectively, where $\rho E_{gas} = \rho E - \rho k$. A failed NEMO/SST state is rejected before conservative clipping or restoration. Convective endpoint checks can separately select a first-order face evaluation when a MUSCL reconstruction is inadmissible.

Each closed stage binds the volume field to a full-precision restart, history, actual configuration, executable, and source identity. Coordinates and all fourteen fields are compared in their corresponding output representation. Independent checks evaluate pressure, translational caloric energy, modal energy, velocity, mixture density, and charge consistency. The explicit distinction between stored-state acceptance and reconstructed-face selection allows stable numerical development and reconstruction behavior to be measured separately.

E. Correction of Turbulence Relaxation Cancellation

The frozen-air SST investigation exposed a specific update mechanism behind an omega plateau. The solver applied a common relative-increment factor to the conservative turbulence increments and then canceled factors below 10^{-10} . At a diagnosed point, ω remained near 10^{-4} while its finite increment required a raw relaxation factor of 6.31946×10^{-13} . Canceling that factor produced a zero update; retaining it produced an admissible increase.

An observation-only build reproduced the baseline residuals and recorded 165 finite-factor cancellations in its first diagnostic call. Of those, 151 accompanied positive omega increments. The repair preserves finite admissible relaxation factors while retaining the existing positivity bounds and relative-update limits. This targets the cancellation rule directly, leaving the continuous SST equations intact.

In a same-start eighty-update comparison, corrected omega RMS was lower by a factor of 9.47657, corresponding to 0.97665 decades. The comparison required identical initial seven-equation right-hand sides, zero reported nonphysical points, complete emitted fields, and converged late linear solves. A separate gradient comparison measured an additional factor of 3.15417 for Green–Gauss relative to corrected weighted least squares at eighty updates. The selected sequence subsequently accumulated 980 updates through its recorded gradient and CFL stages.

The retained trajectory distinguishes the finite-factor correction from later numerical-operator choices. Raising the adaptive CFL ceiling during that historical frozen-air sequence initially increased omega RMS; the latter half and final hundred rows of its selected 500-update segment decreased all seven residuals. The complete frozen-air trajectory eventually reached 40,938 emitted updates. These records provide both the controlled repair result and the broader nonlinear response needed to guide subsequent reacting-flow development.

F. Double-Precision Algebraic Control

The optional graphics preconditioner acts on scaled SST 2×2 diagonal blocks. Native double precision retains the matrix, Krylov iteration, and acceptance calculation. For a candidate solution x , the acceptance quantities are

$$\eta = \frac{\|b - Ax\|_2}{\|b\|_2}, \quad \eta_k = \frac{\|r_k\|_2}{\|b_k\|_2}, \quad \eta_\omega = \frac{\|r_\omega\|_2}{\|b_\omega\|_2}. \quad (27)$$

All three must meet the configured 10^{-7} threshold, with an exact-zero convention for a zero right-hand side. Equation-specific norms keep the much larger omega scale from masking the k-equation solve.

Metal eligibility checks screen the scaled operator and vector ranges. An unsupported application falls back to a complete native double-precision preconditioner evaluation, and flexible GMRES retains the corresponding preconditioned vectors. Execution records distinguish graphics dispatches from processor applications

Table 2 Historical three-update processor benchmark

Threads	Compute, s/update	Speedup	Total, s
1	40.0068	1.000	162.0815
6	13.9988	2.858	61.3213
18	10.7697	3.715	48.8519

Compute speedup uses the one-thread result. Total time covers all three supervised updates, including initialization and output. Checkpoint, executable, configuration, and output schedule were matched; all fourteen saved fields passed the declared comparison.

and record the final true residuals. This policy gives the optional arithmetic path the same native acceptance criterion as the processor control.

The local solver changes also scale the right-hand side by an exact binary power of two, avoiding the generic absolute-epsilon early exit for small nonzero systems. When a provisional solution fails any true-residual threshold, the native correction solves $A\delta = b - Ax$ and updates $x \leftarrow x + \delta$. The original matrix, right-hand side, tolerance, and total iteration allowance are preserved; correction consumes only the remaining allowed iterations. The accepted norms qualify this algebraic direction. Subsequent relaxation and native SST projection have separate nonlinear acceptance through the coupled thermal guard.

V. Retained Donor Development and Flow Assessment

A. Native Parallel Performance

The local workstation supplies eighteen processor cores and 48 GiB of memory. Profiling identified primitive thermochemistry and source evaluation as major costs. Thread-private gas objects and scratch storage allow those pointwise operations to execute in parallel with actual eighteen-worker teams. Edge assembly and species-correction phases retain their recorded serial execution.

Matched three-update runs used identical checkpoints, executables, configurations, and output schedules. At one, six, and eighteen threads, compute times were 40.0068, 13.9988, and 10.7697 s per update. Eighteen threads therefore provided a 3.71477-fold compute speedup over one thread and a 1.29983-fold speedup over six. Total supervised times were 162.0815, 61.3213, and 48.8519 s, respectively. All fourteen single-precision volume components were identical; the maximum component-scaled double-precision restart difference was 4.18547×10^{-17} .

A separate eighteen-thread comparison measured 32.4564 s of compute for the processor preconditioner and 32.9420 s for Metal over three updates. Metal performed six accepted dispatches and 10,749,576 row applications, giving a measured compute overhead of 1.496%. The final volume components remained identical, and the maximum component-normalized restart difference was 1.67419×10^{-16} . The processor path supplies the demonstrated throughput gain; the graphics path supplies an optional, independently controlled application of the same preconditioning operation. These measurements use short matched trials with explicitly separated compute and supervised costs.

B. Current Residuals and Field Development

The retained donor result uses closed refined index 600, with native history through 606. Figures 1 and 2 retain all fourteen component histories, normalized independently by their first value in this continuation. Numerical iteration is steady local pseudo-time. Native history reports 0 nonphysical-point events over the captured record; saved-state thermodynamic acceptance and nonlinear convergence are evaluated separately.

Table 3 compares the complete, coordinate-matched refined fields at 350 and 600. Relative changes use the unweighted nodal Euclidean norm, $\|q_{600} - q_{350}\|_2 / \|q_{350}\|_2$. The residual column uses the native RMS value at those exact indices; the density entry follows the nitrogen-density equation, whose transport dominates mixture density here. The table measures continuing field development directly. Volume-weighted errors, integrated loads, and plateaued engineering outputs require their corresponding diagnostics.

Table 3 Refined field evolution, 350 to 600

Quantity	Nodal change, %	RMS ratio
ρ	6.34536	1.049493
u	5.53127	—
ρE	7.12324	1.115368
ρE_{ve}	7.06950	0.880496
k	32.22712	0.760804
ω	0.04207	0.656857

Table 4 Thermal ranges at refined index 600

Quantity	Minimum	Maximum
T , K	50.011	2355.081
T_v , K	172.747	2340.620
$T_v - T$, K	-1336.608	1083.063

The companion planes, three-dimensional vectors, and integrated streamlines use the same saved native state and verified geometry. Streamlines describe the instantaneous spatial vector field of a steady iterate. The extended wake initially contains the imposed freestream outside the old outlet; its interface with the transferred developing solution is an initialization structure whose evolution is included in the full-field comparison. This coherent set retains 30 primary figures, including detailed shock diagnostics, with the source-bound manuscript revision.

C. Thermal, Chemical, and Charged Species Structure

Table 4 records full-volume temperature ranges at index 600. The mode separation is reported as $T_v - T$, fixing the sign convention across the maps and tabulation. Maximum electron-to-heavy-particle number fraction is 4.6464×10^{-25} ; maximum electron number density is 0.508807 m^{-3} . The maximum oxygen, nitrogen, and nitric-oxide mass fractions are 6.1630×10^{-8} , 1.9672×10^{-13} , and 6.1952×10^{-12} , respectively. Trace quantities are recovered from the double-precision conserved restart without a visualization floor. Their maps represent composition; net production rates require evaluation of the chemical source terms at the same state.

Resolved mass flux is computed from saved conservative momentum on native median-dual boundary areas. The boundary decomposition appears in Table 5. The names inlet, outlet, and side identify geometric roles of zones 3, 4, and 7; all three use the native farfield boundary condition in the executed configuration. Its absolute net-to-incoming ratio is 0.02552565%; the corresponding convective-energy ratio is 0.02547564%. These estimates measure the resolved saved field. An exact discrete audit additionally evaluates the native numerical convective and viscous face fluxes.

Between refined indices 350 and 600, the resolved absolute net-to-incoming mass ratio changes from 0.00163327% to 0.02552565%. Side incoming mass flux changes from 0.103782 to 0.226015 kg/s, while side outgoing mass flux changes from 0.105723 to 0.119616 kg/s. The largest absolute change among these incoming and outgoing boundary terms is the side incoming contribution (+0.122233 kg/s). These changes describe the resolved saved-state flux estimate; nonlinear convergence and exact native numerical-flux closure have separate acceptance tests.

D. Three-Dimensional Compression Measurement

The detailed comparison evaluates 22 probes in each of three states: the first-order coarse-baseline index 980, refined MUSCL index 350, and refined MUSCL index 600. Current pressure-gradient normals determine the probe coordinates; both earlier states are resampled at exactly those coordinates. This removes coordinate drift within the reported comparison. A probe normal can

Table 5 Resolved boundary mass flux at index 600, kg/s

Surface	Incoming	Outgoing
Inlet	399.035275	0.000000
Outlet	0.000000	399.039759
Side	0.226015	0.119616

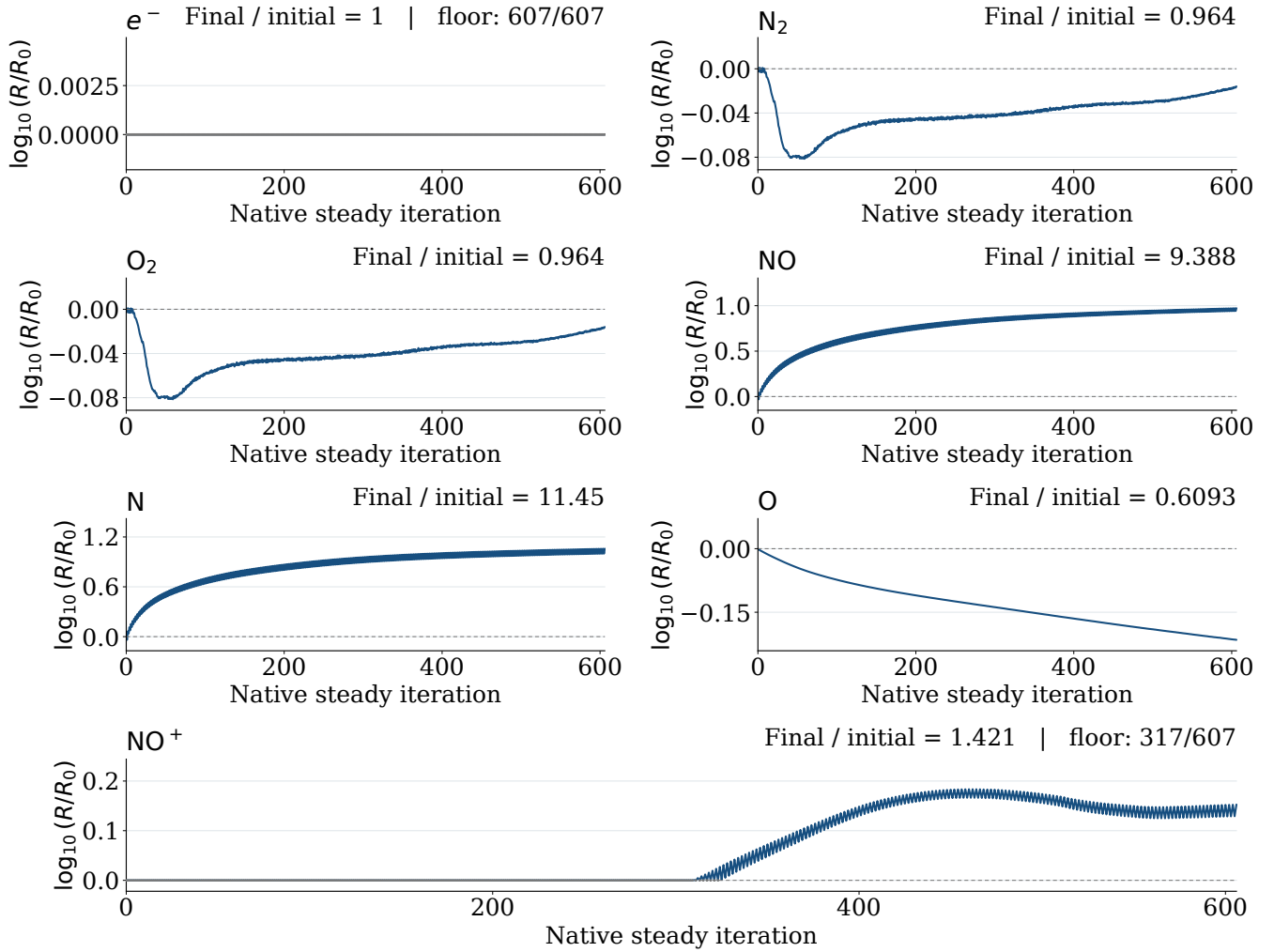


Fig. 1 Historical donor species residuals, showing every native history row from indices 0–606. Each panel reports $\log_{10}(R/R_0)$ for one species. Gray marks retain values reported at the native $\log_{10} R = -32$ floor: 607 of 607 electron entries and 317 of 607 nitric-oxide-ion entries. Values below that reporting floor are not inferred. The retained donor field is index 600; the final history index is 606. No smoothing or sample thinning is applied.

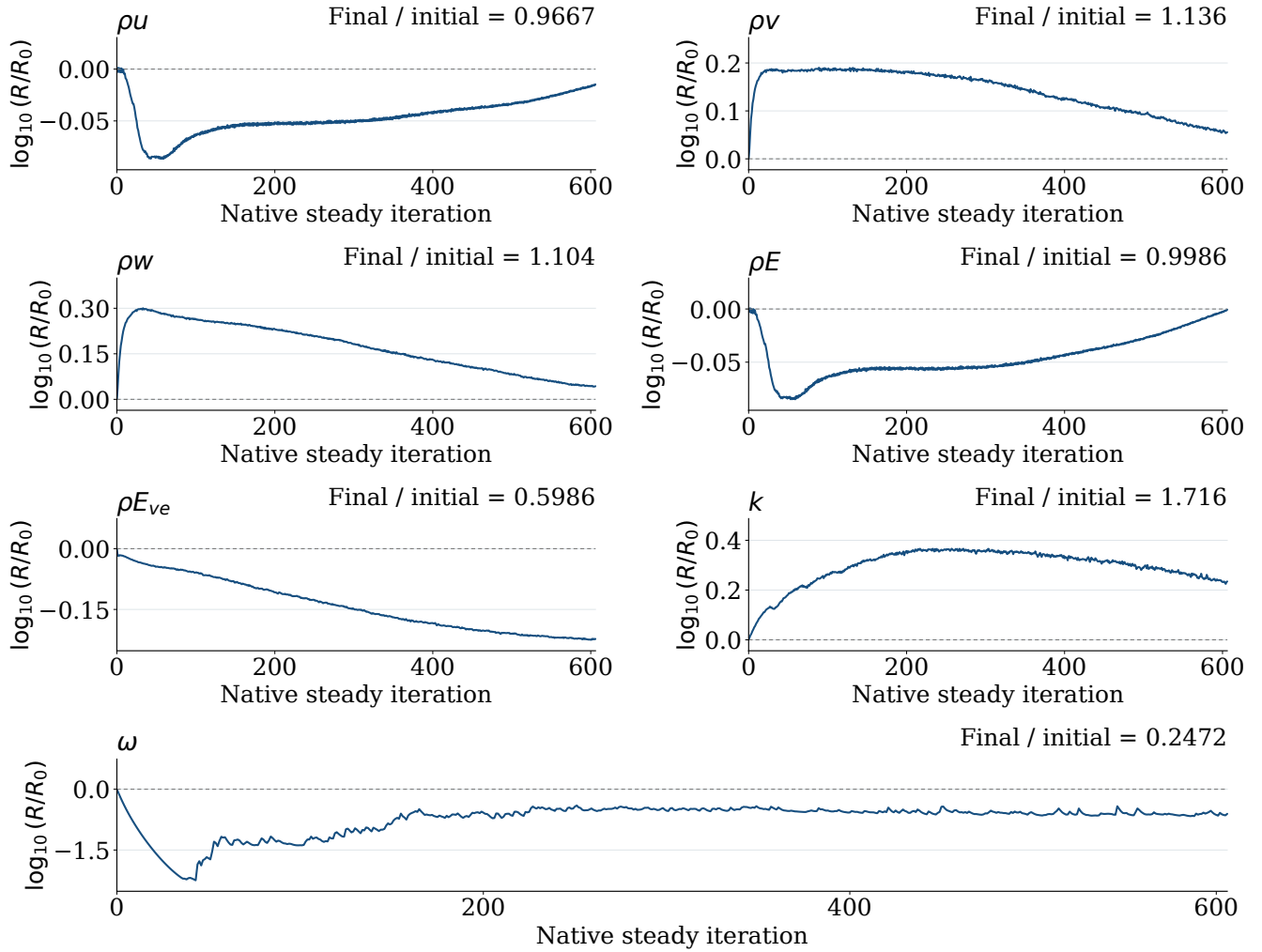


Fig. 2 Historical donor momentum, total-energy, vibrational-electronic-energy, and turbulence residuals, with every native history row from indices 0–606. Separate panels report $\log_{10}(R/R_0)$, normalized by each equation's first reported value. The retained donor field is index 600. These are steady local pseudo-time residual histories; the panels preserve the native samples without smoothing or thinning.

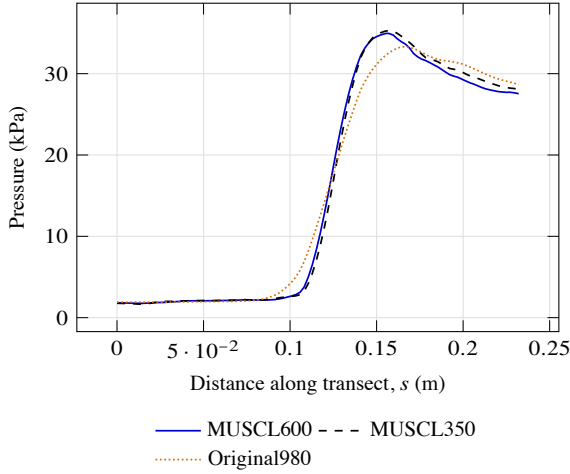


Fig. 3 Intake pressure on identical native-FP64 transect coordinates for refined MUSCL checkpoints 600 and 350, and original first-order checkpoint 980. Each state's independently admitted sample intervals are drawn separately. Compound 10–90 percent pressure-rise widths are 28.955, 28.463, and 43.048 mm, respectively; slashes separate independent admitted intervals when present. These endpoint-to-global-peak widths describe numerical compression; mesh, reconstruction and iteration history vary together in this saved-state comparison.

Table 6 Matching-coordinate compound compression

Quantity	Coarse 980	Refined 350	Refined 600
$p_{\max}/p_u$	17.54549	19.94362	19.09205
$\rho p_{\max}/\rho_u$	2.63655	3.35963	3.24940
$M_{n,u}$	3.80455	3.87961	3.83341
$M_{n,p_{\max}}$	0.14473	0.21089	0.20019
Intake δ_{10-90} , mm	43.0476	28.4630	28.9554
Intake sampled cell hits	10	12	13
Lower δ_{10-90} , mm	113.1145	93.5426	92.9629

change between successive galleries, so measurements from independently oriented older plots are not substituted for these matching-coordinate samples.

For each connected valid interval, the compound measurement uses its upstream endpoint and global pressure maximum:

$$\delta_{10-90} = s(p_u + 0.9\Delta p) - s(p_u + 0.1\Delta p), \quad (28)$$

$$\Delta p = p_{\max} - p_u.$$

The intake pressure and density ratios are 19.09205 and 3.24940; the projected normal Mach estimate falls from 3.83341 to 0.20019. Table 6 reports both endpoint states and the measured widths. The density ratio uses density at the pressure maximum, rather than a separately selected density peak. The 32.74% intake reduction relative to the coarse baseline is a measured sharpening of this developing compression; its change from refined index 350 is +1.7302%. Grid, domain, spatial reconstruction, and iteration history differ across those states; a grid-only convergence rate requires matched independent calculations.

The detailed intake probes use 65 μm sample spacing and linear interpolation at the first upward 10 and 90% crossings. The standard shock overview's intake probe uses 130 μm sampling and its own discrete crossing estimator: its coarse-baseline and donor-600 widths are 43.0300 and 28.9900 mm. These two sampling products remain identified separately. Finer probe spacing samples the same numerical field and creates no additional mesh resolution. The cell-hit row counts distinct native cell identifiers encountered by valid samples inside the interpolated 10–90% interval; unsampled cells are outside that count.

Native Q1 interpolation uses double-precision restart coordinates and conserved density, momentum, and energy. Velocity is reconstructed after interpolation as $u = m/\rho$. Thus the convective normal mass flux is $m \cdot n$, and the normal momentum diagnostic is $(m \cdot n)^2/\rho + p$. The enthalpy diagnostic is $(\rho E + p)/\rho$, including the stored turbulent kinetic energy in ρE . Independently interpolating density and velocity would introduce a product-interpolation

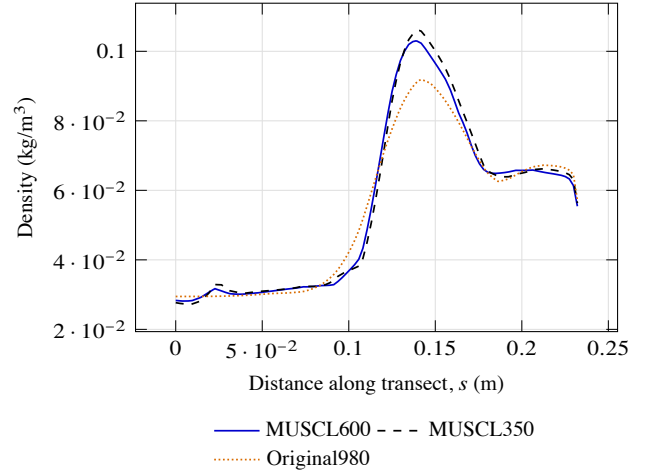


Fig. 4 Intake density on identical native-FP64 transect coordinates for refined MUSCL checkpoints 600 and 350, and original first-order checkpoint 980. Each state's independently admitted sample intervals are drawn separately. Compound 10–90 percent pressure-rise widths are 28.955, 28.463, and 43.048 mm, respectively; slashes separate independent admitted intervals when present. These endpoint-to-global-peak widths describe numerical compression; mesh, reconstruction and iteration history vary together in this saved-state comparison.

Table 7 Intake compound endpoint mismatch, %

Quantity	Coarse 980	Refined 350	Refined 600
$m \cdot n$	-74.0080	-55.4650	-58.8477
$(m \cdot n)^2/\rho + p$	-15.0471	-3.9903	-6.5297
$(\rho E + p)/\rho$	+6.7448	+1.8030	+1.8654

error; the conservative construction avoids it. Pressure and native Mach samples retain their original volume-output precision. Projecting the interpolated Mach onto the conservative velocity direction gives the explicitly approximate M_n used in these figures.

Table 7 evaluates the intake endpoint mismatch as $100(q_{p_{\max}} - q_u)/q_u$. The signed values expose the degree to which the compound endpoint states depart from equal planar inviscid mass, normal momentum, and total-enthalpy values. These endpoints belong to an evolving three-dimensional viscous reacting field. A supersonic-to-subsonic projected Mach crossing and pressure increase identify compression; steady Rankine–Hugoniot acceptance also requires locally appropriate plateau states and flux consistency.

E. Local-Front Acceptance and Sensitivity

The local classifier first detects directional pressure-gradient peaks above the greater of 2% of the interval maximum and the interval pressure-noise scale divided by its length. The noise scale is 8 single-precision machine epsilons times the maximum absolute interval pressure. Each principal gradient core is the contiguous region around its peak at or above 20% of that peak derivative. Compression requires net pressure and density increase, normal deceleration, and simultaneous positive density gradient and negative normal-velocity gradient over at least 50% of core samples. The independently evaluated native $u \cdot \nabla p$ must be positive over at least 90% of core samples. The retained supported candidates include gradient-core sensitivity at 10, 20, 30% of peak derivative.

Upstream and downstream plateaus each require 2 adjacent complete native-cell chords, relative pressure/density/velocity spread no greater than 2%, and relative linear trend no greater than 2%. Their pressure derivative must remain below 5% of the candidate peak; velocity directions must remain within 5° of the plateau mean. The pressure connection to the core must be monotone within the recorded precision allowance, without skipping an intervening feature. Local pressure-gradient alignment and the local-to-probe normal angle must satisfy 25°. An accepted jump additionally has increasing pressure and density and positive, decreasing normal velocity between the plateaus.

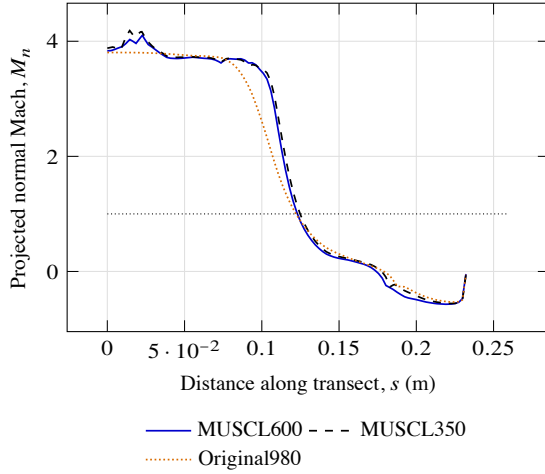


Fig. 5 Intake normal Mach on identical native-FP64 transect coordinates for refined MUSCL checkpoints 600 and 350, and original first-order checkpoint 980. Each state's independently admitted sample intervals are drawn separately. Compound 10–90 percent pressure-rise widths are 28.955, 28.463, and 43.048 mm, respectively; slashes separate independent admitted intervals when present. These endpoint-to-global-peak widths describe numerical compression; mesh, reconstruction and iteration history vary together in this saved-state comparison. $M_n = M(u \cdot n)/|u|$ is an approximation using interpolated native Mach and velocity reconstructed as interpolated conservative momentum divided by interpolated density. The dotted line marks unity.

Table 8 Local-front classifications over the matched probes

Classification	Coarse 980	Refined 350	Refined 600
Mixed alignment	28	36	42
Opposed/transverse	18	31	36
Insufficient compression	7	16	15
No downstream plateau	5	16	10
Qualified plateau jump	0	0	0

Table 8 counts classifications across all declared probes. A feature intersected by several probes produces several detections; these counts do not enumerate distinct physical shocks. At index 600, 10 candidates pass the preliminary compression/alignment tests; 10 are retained for detailed plateau tests, with at most 3 supported candidates per probe. The retained and rejected records contain 103 classified detections, including 0 accepted isolated plateau jumps. Gradient cores and endpoint-to-peak compound widths retain their numerical definitions, distinct from physical shock thickness.

The principal core is selected by the largest positive sampled directional pressure derivative among all recorded candidates on each principal probe, including rejected candidates. Table 9 records its width, peak derivative g_p , compression-support fraction f_c , and positive-work fraction f_w . At donor index 600, their classifications are intake: mixed alignment; lower: mixed alignment. This explicitly tests the strongest pressure features rather than interpreting every steep contour as an isolated accepted shock.

The measured normal-offset experiment evaluates nearby native pressure-gradient normals at offsets ranging from 10 to 1000 μm . Table 10 gives their angular changes relative to the anchor normal. It measures local orientation sensitivity of the sampled saved field; compound-width sensitivity to reoriented probes requires separately resampled lines. The detailed atlas also retains spanwise and transverse probes, pressure/density gradients, divergence, vorticity, and thermal response.

Point admissibility is checked independently of VTK line traversal using the native hexahedral inverse map, finite shape weights, a 10^{-9} weight-sum tolerance, and minimum weight -10^{-7} . A rejected interpolation sample is recorded and excluded. Traversal

Table 9 Principal 20%-gradient cores at index 600

Probe	Width, mm	g_p , MPa/m	f_c , %	f_w , %
Intake	39.455	1.161294	80.10	87.17
Lower	106.665	0.081383	90.38	87.09

Table 10 Measured native-normal offset sensitivity, degrees

Anchor	Samples	Median	Maximum
Intake	12	0.62367	2.19146
Lower	12	0.04963	1.98832

Table 11 Center-plane wake Mach ranges at index 600

x interval, m	$M_{\min}$	$M_{\max}$	Points
2.9–3.5	0.00000	6.27813	4955
3.5–4.30553	2.35797	6.36378	1962
4.31–5	2.55152	6.10564	1512
5–6	2.55403	6.10564	2142
6–7	3.12926	6.73030	2142
7–8	5.99122	6.01475	2079
8–8.7	5.99994	5.99996	1449

gaps and overlapping intervals describe limitations of that traversal, without asserting physical gaps or overlapping mesh cells. Consequently, sampled cell hits are reported separately from an exhaustive cell count.

F. Domain Extent and Developing Wake

The saved native domain extends to $x = 8.7$ m, beyond the original outlet at $x = 4.305532$ m. Table 11 samples the $y = 0$ plane within $-0.5 < z < 0.5$ m. Each row reports the actual retained section-point extrema and sample count over its declared axial interval. These measurements locate variation along the developing extended wake at this steady iterate. They characterize the stated center-plane strip; complete cross-sectional wake integrals require integration over the corresponding native three-dimensional sections.

G. Localized Upstream Quality Assessment

The fixed intake window $0.1 < x < 0.23$ m, $0.02 < z < 0.15$ m at $y = 0$ compares the refined and coarse-baseline section-point extrema in Table 12. The cold, low-pressure, high-Mach region precedes the principal pressure rise. The tabulated observations identify a specific region for numerical and physical diagnosis; attributing its cause requires a controlled investigation of reconstruction, coupling, initialization, and local geometric resolution. This concern is retained alongside the measured downstream sharpening.

The gradient-map display is checked independently of these field extrema. Of 70571 display-section triangles, 2476 have exactly zero pressure-gradient magnitude and 0 have exactly zero density-gradient magnitude; all assessed values are finite. A logarithmic color normalization previously masked the zero pressure-gradient values and produced blank wedges. The corrected display maps zero and positive subthreshold gradients to the colormap's lower endpoint, preserving the raw gradients and native geometry. This display correction explains those blank wedges. The independently measured low-pressure, cold, high-Mach region remains a field-quality observation requiring its own diagnosis.

H. Three-Dimensional Field and Native-Grid Atlas

The publication atlas shows the native hexahedral grid, vehicle surfaces, intake, and extended wake. Figure 40 displays the geometric allocation and selected native edges. Figures 46 and 47 separate dense streamlines from magnitude-scaled vectors for readable directions and local speeds. Their source records specify seeds, vector scale, integration settings, and termination statistics.

A streamline is a spatial integral curve of the saved velocity field,

$$\frac{dx}{ds} = \frac{u(x)}{|u(x)|},$$

where s denotes arc length away from stagnation. The curves

Table 12 Upstream-window section extrema

Quantity	Coarse 980	Refined 600
$p_{\min}$, Pa	1897.3186	536.9546
$T_{\min}$, K	223.4745	61.6348
$M_{\max}$	6.0000	11.9002

characterize the retained donor steady iterate; integration accuracy, native-cell validity, and sampling density are assessed separately from CFD convergence. Conservative mixture velocity equals sampled momentum divided by sampled density; the rendering record distinguishes this from interpolated nodal velocity. Native validity masks retain solid and exterior gaps.

Figures 41 and 42 place both mode temperatures and their signed difference in the same geometry. Figures 43 and 44 display trace composition with declared logarithmic color ranges. Electron number density and electron-to-heavy-particle number fraction use

$$n_e = N_A \frac{\rho_e}{M_e}, \quad x_e = \frac{\rho_e / M_e}{\sum_{s \neq e} \rho_s / M_s},$$

with molar masses in kg/mol. Color encodes composition, not luminosity or chemical production rate. Pressure level sets and probes in Fig. 45 locate three-dimensional compression; measured gradients, conservative flux profiles, and local-front tests govern their interpretation.

I. Detailed Compression and Quality Atlas

The ten diagnostic plates after the references retain matched native-grid sections, both principal conservative profiles, the center-plane transect, compression, vorticity, temperature response, normal perturbations, spanwise sensitivity, and upstream-window extrema. Comparisons use declared coordinates and scalar scales. Separate pressure and density derivatives expose their geometric coincidence and compressive support at the same spatial scales as the compound-width measurements.

VI. Donor-Grid Refinement and Its Verification

A. Inverse-Map Selection and Sheet Closure

The refinement algorithm combines saved pressure information with the local hexahedral map. Let $x(\xi)$ be the trilinear map from the reference cube and $J = \partial x / \partial \xi$. For a physical probe direction n ,

$$\frac{d\xi}{ds} = J^{-1} n. \quad (29)$$

Its component magnitudes indicate how quickly the probe crosses each logical coordinate. This connects a measured physical compression direction to subdivisions that add useful cell crossings on a skewed grid.

The source field is the closed original-grid MUSCL index 320. Native Q1 center derivatives and double-precision coordinates define

$$\eta_c = \frac{h_n |\nabla p|}{\max(\bar{p}, p_\infty)}, \quad h_n = \frac{2}{\sum_a |\nabla N_a \cdot n|}, \quad (30)$$

where N_a are the eight trilinear shape functions and $n = \nabla p / |\nabla p|$. The selection requires $\eta_c > 0.1$, $\nabla \cdot u < 0$, and $\bar{M} > 0.5$, marking 36,099 original cells. The wake corridor downstream of the aftmost wall is $r < 0.65 + 0.05 \max(x - 4.305532, 0)$ m; admissible directions with axial span greater than 0.04 m identify 140,362 wake seed cells.

The inverse-map selection requires useful normalized directional projection of at least 0.1. All admissible axes on the fixed intake/lower probes and all requested wake directions are mandatory. Wall-adjacent normal-axis exclusions propagate through conformity closure to five connected topological sheets. They preserve the original first-wall spacing; five sheets do not mean five wall-normal layers. Remaining sheets are prioritized by pressure-sensor and inverse-map crossing benefit per exact added cell.

Conformity is enforced through topological edge sheets. Each local hexahedral axis groups its four parallel edges. Shared edges connect compatible subdivision decisions even when neighboring cells use rotated local axes. Propagating a split through this graph establishes matching shared-face subdivisions. Original point identifiers remain a prefix, and new edge midpoints, face centers, and cell centers are deduplicated by their topological entities.

The graph contains one vertex for each cell-axis and connects axes that share an undirected topological edge. Connected compo-

Table 13 Original and selected refined geometry

Quantity	Original	Refined
Hexahedral cells	1,742,500	3,998,832
Points	1,791,596	4,071,845
Outlet x , m	4.305532	8.700000
Outer radius, m	0.774047	1.548094
Intake-interval cells	10	19
Lower-probe cells	12	23

nents are conforming split sheets, accommodating rotated local axis labels across neighboring cells. If $r_c(\mathcal{S})$ axes of cell c are selected by sheet set $\mathcal{S}$, the exact cost and marginal cost are

$$N_c(\mathcal{S}) = \sum_c 2^{r_c(\mathcal{S})}, \quad (31)$$

$$\Delta N_\sigma = N_c(\mathcal{S} \cup \{\sigma\}) - N_c(\mathcal{S}).$$

Marginal costs are recomputed after each choice, including repeated sheet labels within a cell. Benefit-per-cost ordering uses stable identifier ties, making the selection reproducible. An unrestricted closure grew the expanded mesh beyond eight million cells; measured workstation capacity supported the selected approximately four-million-cell configuration. This is offline sensor-guided adaptation with an explicit engineering allocation rule; it is not an online error-equidistribution algorithm.

An initial physical-edge projection criterion increased the intake interval from ten to eleven cells. The inverse-map and mandatory-probe revision increased it to nineteen. The final allocation selects 134 of 763 sheets and covers 62.15% of preferred shock axes, 64.38% of their weighted sensor benefit, and all requested wake axes. Coverage measures the specified selection objective, independently of subsequent flow accuracy.

B. Complete-Cell Geometry Verification

The domain extension first extrudes the outer quadrilateral shell radially, then extrudes the complete expanded outlet axially using shared rim nodes. Nominal layer growth is 1.1, rescaled to hit the target extent without a thin terminal layer. Twenty radial and seventy-seven downstream layers expand the outer radius to 1.548094 m and move the outlet to $x = 8.7$ m, providing 5.717416 m behind the aftmost wall. Table 13 summarizes the selected mesh and fixed-probe cell counts.

The intermediate outer and wake extension contains 2,302,420 hexahedra before directional refinement. Table 13 retains the exact baseline and final donor mesh counts, physical extents, and probe allocations.

Verification checks internal-face pairing, opposite winding, exact external marker coverage, original-coordinate preservation, and exported-mesh readback. Outward-rounded interval Bernstein bounds certify positive Jacobians over every complete trilinear reference cell, rather than only sampled corners or Gauss points. All 3,998,832 cells pass, with a minimum certified bound of $4.62550 \times 10^{-17} \text{ m}^3$. This is a local whole-cell and connectivity certificate; arbitrary intersections between non-neighbor cells are outside the executed geometric test.

Parent-volume comparison retains a relative screening threshold of 2×10^{-10} . Twenty thin-cell parents require further analysis because rounded subdivision coordinates amplify relative differences. Exact-rational integration verifies ideal Q1 subdivision volumes, then determinant-product bounds propagate the permitted coordinate-rounding error. The maximum exact relative volume difference is 3.76521×10^{-10} , within its derived rounding bound. This resolves the arithmetic mechanism while preserving the original screening tolerance.

The intake count refers to the old 10–90% pressure-rise interval; the lower count refers to the complete fixed lower transect. Original wall-normal spacing is retained, while inserted tangential nodes supply additional sampling. These allocation counts remain distinct from later sample-hit counts along evolving pressure-normal probes. Tests exercise rotated neighboring cells, every directional split pattern, shared-node identity, forbidden-sheet and budget re-

fusals, warped cells, thin-cell rounding, and rejection of genuine coordinate perturbations.

C. Steady Initial-Guess Transfer

A deterministic warm start carries the developed coarse MUSCL state at index 320 onto the refined mesh. All fourteen stored fields at the 1,791,596 original nodes are preserved bitwise. Added domain nodes in the expanded-grid prefix receive the uniform native freestream state. New refinement nodes use the exact topological Q1 map: the twelve conserved flow quantities are interpolated directly, while ρk and $\rho \omega$ are interpolated and divided by the interpolated density. Total energy retains its turbulent-energy contribution. This construction supplies a nonconservative steady initial guess; physical-time histories and domain-integrated inventories are outside its transfer contract.

Before native execution, source-backed caloric and two-temperature pressure checks verify admissibility, bracket the modal-energy inversion, and enforce quasineutrality. Geometry, wall ancestry, no-slip velocity, and the native wall- k zero or floor convention are checked independently. Subsequent native saved-field verification tests the complete fourteen-field restart against volume output, including recovered temperatures, pressure, density, velocity, and charge.

VII. Historical Resource Qualification and Computational Sequence

The refined connectivity requires 2,302,240,520 bytes for species-correction edge and point storage. A bounded configuration parameter replaces the former fixed 2-GiB allowance with a default of 2048 MiB and a strict maximum of 4096 MiB. The selected case uses 3072 MiB. Parsing, arithmetic overflow, and exact required-byte checks remain active. The resource-only helper passes 621 sanitizer-backed checks.

Native zero-update initialization measured 38.576 s and a peak resident memory of 18.72 GiB. The subsequent three-update cold-start pilot completed in 184.060 s, including preprocessing and full-state output, with a peak of 21.55 GiB. Complete saved-field checks passed, the maximum charge-relative error was 2.49464×10^{-16} , and the maximum native global/ k /omega solve ratio was 5.97960×10^{-11} . Actual pointwise processor teams contained eighteen workers.

The pilot performed eleven processor preconditioner applications and zero graphics dispatches under the declared preference policy. Its final assembly flagged 3,490,679 reconstructed endpoints, approximately 14.37% of twice the refined grid's 12,142,150 edges. The refined and original figures describe different developing states; both are retained with their initialization and counter definitions. This early refined calculation on 4 October 2026 started from freestream, followed by continuation from its own exact fourteen-field restart.

Learned-initialization experiments define a candidate-generation interface followed by native correction and acceptance. In the autonomous frozen-air rollout, the second step produced twelve nonpositive-density and ninety nonpositive-pressure nodes. A steady learned warmstart raised imported-state residuals by approximately 2–4%. The retained one-step 40-to-50-ns comparison improved on persistence for all five conserved components. A separate matched full-domain inference test measured 4.93378 s on eight processor threads and 1.13519 s on the graphics tensor path, with relative output difference 4.84716×10^{-8} . The record consequently reports predictive behavior and execution cost as distinct measured quantities.

The computational sequence advances through explicit measured interfaces: diagnosis, qualified state, compression measurement, geometric allocation, full-cell certification, and native resource admission. Closed configurations, executable hashes, fields, restarts, and histories preserve each transition. The early freestream-initialized continuation on 4 October 2026 completed fifty-two updates and retained its final field at index 51 after a memory-pressure termination. Its peak resident memory was 22.75 GiB. The closed

field passed the same fourteen-state, caloric, pressure, and charge checks and remains a separately retained initialization experiment.

Later on 4 October 2026, the coarse-state warm start passed native readback and three successive bounded stages at nominal CFL values of 0.05, 0.1, and 0.5. Each stage completed three updates, reported zero nonphysical points in its history, and passed complete saved-field checks. Each continuation used the preceding accepted native restart. At CFL 0.5, the SST linear-iteration allowance increased from five to twenty while the global and equation-specific 10^{-7} acceptance thresholds remained fixed. The largest accepted k -equation ratio in that stage was 3.83972×10^{-9} . The final nominal CFL is 500 times the original value of 0.001. This ratio describes the configured pseudo-time control; the bounded stages establish restart and update admissibility, while throughput gain and steady convergence require their own measurements.

The subsequent run033 stopped during its fifth attempted update, after four history rows: the native energy-admissibility check rejected an out-of-range translational temperature while memory pressure was normal. The final meanflow thermal restriction addresses that candidate mechanism. Run035 subsequently exposed the distinct post-SST loss of translational energy at fixed flow total energy, motivating the coupled SST thermal restriction. Both failures and their exact diagnostic states are retained.

With both guards integrated, run037 completes ten native updates at CFL 0.5, crosses both former failure points, and passes complete fourteen-field verification. Its final checkpoint initialized the subsequent donor run038. An inherited point-loop chunk, previously left unset because NEMO bypasses the generic hybrid initializer, is initialized deterministically to 512 for the eighteen-worker point loops. Matched run036/037 checkpoint5 has identical seventeen-column restart data, including coordinates. Median primitive-phase time decreases from 14.28917 to 1.13478 s, a factor of 12.59206 for that phase. Edge assembly remains serial, so this phase ratio is not a whole-solver speedup.

The historical donor continuation used the verified run037 executable, explicit flow, implicit SST, nominal CFL 0.5, and full-field/restart output every fifty iterations plus final output. Its captured history, linear-direction checks, thermal restriction counts, and field results accompany the retained donor figures. The graphics dispatch count in that donor native record was zero; reacting thermochemistry and flow executed on the processor. The later fine-grid backend is described separately below. The optional graphics preconditioner and learned-model benchmarks above retain their separately measured scopes.

VIII. Fine-Grid Native Backend and Coarse-to-Fine Initialization

A. Conserved State and Active Coupling Contract

The fine-grid calculation advances the existing reacting AIR-7, two-temperature, SST system through a bounded-memory native backend. The seven species are ordered as e^- , N_2 , O_2 , NO , N , O , and NO^+ . At a global node, the stored fourteen-component state is

$$\mathbf{Q} = (\rho_0, \dots, \rho_6, m_x, m_y, m_z, q_E, q_v, k, \omega)^T, \quad (32)$$

$$\rho = \sum_{s=0}^6 \rho_s, \quad \mathbf{u} = \mathbf{m} / \rho.$$

The first twelve entries are conserved densities; the last two are specific SST variables. In particular, their storage as k and ω does not change the density factors in the native scalar update. Total energy includes turbulent kinetic energy:

$$q_E = \sum_s \rho_s e_{tr,s}(T) + q_v + \frac{|\mathbf{m}|^2}{2\rho} + \rho k, \quad (33)$$

$$q_v = \sum_s \rho_s e_{v,s}(T_v).$$

The modal energy includes molecular vibration, electronic excitation, and electron translation. Heavy-species formation offsets belong to $e_{tr,s}$; electron formation and translation belong to $e_{v,e}$. The native pressure is $p = AT + BT_v$, where $A = \sum_{s \neq e} \rho_s R_s$ and

$B = \rho_e R_e$. With $C_{tr} = \sum_{s \neq e} \rho_s c_{v,tr,s}$ and $C_v = \sum_s \rho_s c_{v,v,s}$, the executed frozen acoustic closure is

$$a^2 = \frac{p_h + p_e}{\rho} + \frac{p_h A}{\rho C_{tr}} + \frac{p_e B}{\rho C_v}, \quad p_h = AT, \quad p_e = BT_v, \quad (34)$$

with the last term evaluated when electron density is positive. A fixed perfect-gas ratio of specific heats is therefore unnecessary for recovery or visualization.

The backend calls the native thermochemistry [4, 12], AUSM⁺-up2 flux [8, 9], Green–Gauss gradients, Venkatakrishnan–Wang reconstruction, viscous transport, chemical sources, and SST routines [6, 7]. Its changes concern storage, ownership, scheduling, exact reuse, and initialization. The inherited two-temperature modifications remain explicit: $D_t = \mu_t / (\rho S c_t)$; separate turbulent conductivities use $Pr_t = 0.9$; species transport uses $Sc_t = 0.7$ and a mass-preserving, zero-current charged-pair projection. The ion molar mass is $M_{NO^+} = M_N + M_O - M_e$. The modal source remains

$$S_v = \sum_{s \neq e} \frac{\rho_s}{\tau_s} [e_{vib,s}(T) - e_{vib,s}(T_v)] + \sum_s \dot{\omega}_s e_{v,s}(T_v) - p_e \nabla \cdot \mathbf{u}. \quad (35)$$

Here the relaxation difference uses vibrational energy alone, whereas chemical production transports full modal energy. The inherited reaction-temperature regularization, finite-rate tables, and heavy-particle relaxation remain active. This closure supplies quasineutral reacting-air transport; it contains neither a resolved electric-field equation nor a separate electron-heavy collisional exchange model. The SST stress uses the deviatoric effective-viscosity closure. The retained total-energy model includes ρk but omits modeled k diffusion and the isotropic $-2\rho k \mathbf{I}/3$ stress contribution, as stated in the governing-equation formulation.

B. Global Phase Barriers with Bounded Native Geometry

The receiver contains 42,396,569 nodes, 41,962,496 hexahedra, and 126,754,904 native edges. Sixty-four ownership partitions retain the original global identities and native geometry. Each partition constructs the support required by the original nodal and edge operators; only designated owners publish global rows. Artificial partition boundaries do not receive physical boundary conditions. The geometry is still processed by SU2's unstructured finite-volume connectivity, while every original hexahedron is retained. No downstream or radial extension from the coarse donor is appended to this receiver.

One epoch is one complete globally coupled steady update. Its execution first reconstructs the accepted FLOW state with globally consistent limiter extrema and numerical carry. Native convective and viscous edge records are evaluated in bounded batches. These records enter the assembled residual in the canonical global edge order, followed by sources, far-field contributions, and strong wall operations. For an oriented interior edge ij , the spatial assembly retains the opposite endpoint contributions

$$\mathbf{R}_i += \mathbf{F}_{ij}, \quad \mathbf{R}_j -= \mathbf{F}_{ij}. \quad (36)$$

For positive viscous-flux convention $\mathbf{F}_s = -\mathbf{J}_s$, the viscous contribution enters the residual with the opposite sign to convection. The species correction uses one face fraction for all seven species and the corresponding total/modal energy fluxes. Thus the transport assembly retains its pairwise spatial conservation when its evaluation is partitioned.

After the complete FLOW residual is available, a single local pseudo-time increment advances each node's twelve-component FLOW block. The accepted configuration fixes nominal CFL at 0.5; source-aware species depletion and thermal admissibility reduce individual increments. Species depletion leaves a ten-percent inventory margin in the low-order candidate. Following flux correction, the thermal search holds the residual fixed and halves the common local increment until both recovered temperature bounds and the modal caloric bracket are satisfied. The update is

$$\mathbf{U}_i^* = \mathbf{U}_i^n - \frac{\Delta \tau_i}{V_i} \mathbf{R}_i^n, \quad \mathbf{U}_i = (\rho_0, \dots, \rho_6, \mathbf{m}, q_E, q_v)^T. \quad (37)$$

The electron/ion pair is updated through its combined mass coordinate and expanded at the fixed molar-mass ratio. These safe-

guards preserve the declared closure without trace-species floors. Spatial conservation of Eq. (36) and a spatially varying $\Delta \tau_i$ describe a steady relaxation method; epoch count is not elapsed physical time. The executed configuration does not contain a residual-driven adaptive-CFL controller.

A global barrier closes FLOW before velocity gradients, strain, vorticity, and SST gradients are recovered. The SST system is assembled into one global sparse matrix, with its native 2×2 block structure. Partition matrices are assembly workspaces; no independent partition linear solves replace the global problem. The native FGMRES direction $\delta \mathbf{z}$ solves $\mathbf{A} \delta \mathbf{z} = \mathbf{b}$. Its global, k , and ω true FP64 residual ratios must each be at most 10^{-7} . The subsequent nonlinear SST update is

$$q_i^+ = \Pi_q \left[\frac{\rho_i^* q_i^n + \alpha_i \delta(\rho q)_i}{\rho_i^*} \right], \quad q \in \{k, \omega\}, \quad (38)$$

where Π_q is the existing native component bound. The common relaxation factor is further reduced if the projected k would violate the thermal state at fixed q_E and q_v . The solved-direction criterion precedes this relaxation and projection. Once all SST owners are updated, a second global gradient barrier and native postprocessing recover the final transport state. This sequence prevents old halo values or partially written turbulence from entering a published viscosity field.

C. Exact Reuse and Heterogeneous Execution

The measured host provides 48 GiB of unified memory and eighteen physical CPU cores in two classes, containing six and twelve cores. The operating system reports eighteen logical cores, 128-byte cache lines, and 16 KiB virtual-memory pages. Per-level reported L1 data-cache sizes are 128 and 64 KiB; reported L2 sizes are 16 and 8 MiB, respectively. These are the operating system's level descriptors, not a summed chip-wide cache capacity. The active implementation requests eighteen CPU threads. Thermodynamic recovery, reaction and relaxation rates, native flux evaluation, gradients, and the global SST Krylov solve execute on the CPU. GPU work consists of ordered FLOW residual replay and selected flux-correction passes. The selected GPU kernels use integer-based software binary64 addition, subtraction, multiplication, and comparison from the embedded `metal-softfloat` library [27]. This implementation integrates those existing arithmetic primitives into ordered FLOW replay and flux correction. Thermochemistry and the SST solve retain native CPU double precision. Within each replay batch, GPU and CPU workers own disjoint residual rows, process incident edges in the same order, and synchronize before the assembled rows are published. The configured twenty-five-percent GPU row allocation is a work partition, not a GPU-utilization target.

Shared addressable buffers avoid a discrete-device transfer protocol, but this implementation still packs records, builds local incidence, gathers rows, and scatters completed values. These operations are timed separately. Reported GPU activity therefore refers to executed commands and affected rows, rather than inferred acceleration from a shared-memory architecture. Metal shader compilation disables fast math; the native build uses strict floating-point semantics with contraction disabled.

Several changes reduce repeated work while preserving dependencies. Convective and viscous records are evaluated during a common native tile visit, then replayed in their original phase order. Boundary operands are captured during source visits and replayed after all source contributions are complete. Far-field convection and viscosity remain two separate FP64 additions; strong wall rows are imposed after the far-field phase. Retained characteristic states supply the subsequent SST boundary calculation. This preserves rounding order even where marker operations affect the same node.

The topology cache holds verified immutable connectivity, with an admitted 4,096 MiB capacity; the measured full-grid contents occupy 4,043,201,880 bytes. Each typed record store has a separate 2,048 MiB cache allowance. The fused staging phase holds convection and viscous stores simultaneously, allowing up to 4,096 MiB combined for these two caches. Ordered sealing combines adjacent

dirty pages into bounded vector writes. Limiter reuse is bound to the accepted state, numerical carry, global extrema, and reconstruction settings. Thermochemical memoization keys use the exact temperature bit pattern, mode-selection flag, floating-point control, and model data; approximate temperature binning is absent. A cached value therefore reproduces the native computation for the same inputs.

The SST solve also reduces retained Krylov storage. With a fixed native CPU Jacobi map $\mathbf{P}^{-1}$, the implementation retains Arnoldi vectors $\mathbf{v}_j$ and reconstructs

$$\mathbf{z}_j = \mathbf{P}^{-1} \mathbf{v}_j, \quad \delta \mathbf{z} = \sum_j y_j \mathbf{z}_j \quad (39)$$

when forming the correction. It verifies the inverse-block generation and fingerprints of the original and reconstructed directions. The original four-vector accumulation grouping is retained. For a twenty-vector subspace, eliminating twenty stored two-component preconditioned vectors saves $20(2N)(8)$ bytes, or 12.635 GiB on this grid. The global matrix and true-residual checks remain intact. This optimization requires a fixed CPU preconditioner; it is not applied to a variable GPU preconditioner.

D. Nonmatching-Grid Initial-Guess Transfer

The donor is the developed checkpoint-600 solution on 4,071,845 nodes and 3,998,832 hexahedra. Its flow supplies a nonlinear initial guess for the unchanged fine receiver mesh, following the established coarse-to-fine initialization principle [13]. A bounding-volume hierarchy selects candidate donor cells, and a safeguarded Newton inversion locates each receiver node in the donor's native trilinear hexahedron. For natural coordinates $\xi = (\xi, \eta, \zeta)$ and corner bits $\mathbf{b}_a \in \{0, 1\}^3$, define

$$N_a(\xi) = \prod_{d=1}^3 [b_{ad}\xi_d + (1 - b_{ad})(1 - \xi_d)], \quad \mathbf{x}(\xi) = \sum_{a=1}^8 N_a \mathbf{x}_a. \quad (40)$$

Only admissible containing-cell coordinates are used. The lookup imposes both a Cartesian error bound $10^{-12} \text{ m} + 10^{-11} d_B$, where d_B is the donor bounding-box diagonal, and a row-scaled inverse-map error bound of 10^{-9} . Natural-coordinate excess is limited to 10^{-9} ; any permitted clipping is followed by the same geometric checks. Shared faces select a deterministic donor identity. Multiple strict cell interiors are rejected. The mapper verifies positive Jacobians at cell centers and accepted sample locations; whole-element isoparametric invertibility requires stronger conditions [17], and the donor's separate mesh-quality evidence supplies the broader geometric context.

At a located receiver point, the twelve FLOW densities are interpolated directly, while the specific turbulence variables are density weighted:

$$\begin{aligned} \mathbf{U}^h &= \sum_a N_a \mathbf{U}_a^H, & \rho^h &= \sum_s \rho_s^h, \\ k^h &= \frac{\sum_a N_a \rho_a^H k_a^H}{\rho^h}, & \omega^h &= \frac{\sum_a N_a \rho_a^H \omega_a^H}{\rho^h}. \end{aligned} \quad (41)$$

Consequently $\rho^h k^h = \sum_a N_a \rho_a^H k_a^H$, consistent with the turbulent-energy content already present in q_E^h . Adding another $\rho^h k^h$ to the transferred total energy would count that contribution twice. Non-negative shape functions preserve nonnegative species and the linear quasineutral relation. Exact coincident corners copy all fourteen donor components bit for bit. Pressure and temperature are subsequently recovered from the native caloric model, rather than interpolated independently.

The pointwise transfer preserves interpolated conserved variables, but does not impose equality of coarse- and fine-grid volume integrals. It is a nodal initial-guess construction; integral-conservative transfer instead requires a corresponding projection or overlap formulation [15, 16]. The target far-field pressure remains 2027.815859879755 Pa, while the donor was calculated at approximately 1897.2735 Pa. No blanket pressure or energy scaling is applied. The new boundary-value problem relaxes from the transferred field under its own original boundary conditions; transferred

compression structures are initial data rather than an already converged fine-grid solution.

E. Coverage, Fine-Wall Recovery, and Initial-State Qualification

The final lookup locates 41,806,828 receiver nodes, or 98.6090%. For the remaining 589,741 nodes, the construction retains the exact previously accepted fine-grid checkpoint-76 state at the same global node identity. Full coordinate and connectivity comparisons establish that this fallback uses the same receiver geometry. It introduces no nearest-point extrapolation or cross-wall interpolation. The fallback accounts for 203,653 native wall nodes, 41 upstream boundary nodes, and 386,047 interior nodes in wall-connected patches. There are no detached interior patches in that accounting. Its summed native dual-volume fraction is 9.83912×10^{-6} . Independent local geometric checks place the seventy-four deeper wall-connected review points outside the donor fluid region. Individual causes of every shallow rejection were not separately resolved; exact accepted fine values provide the explicitly bounded initialization policy for those nodes.

The native initialization utility then enforces the receiver's stationary noncatalytic adiabatic walls. For each fine wall node it applies

$$\mathbf{m}^w = \mathbf{0}, \quad k^w = 0, \quad q_E^w = q_E - \frac{|\mathbf{m}|^2}{2\rho} - \rho k, \quad q_v^w = q_v, \quad (42)$$

while preserving all species densities. Thus the prescribed wall adjustment removes mean and turbulent kinetic energy while retaining thermal/modal content. The complete wall correction is closed globally before any subsequent gradient or transport recovery. The native SST wall routine sets

$$\omega_w = \frac{60\mu_w}{\rho_w \beta_1 d_w^2}, \quad (43)$$

using the receiver-mesh nearest-interior distance d_w , not a transferred coarse wall distance. This matters strongly because the wall value varies as d_w^{-2} .

After every wall value is available, global native Green-Gauss SST gradients are reconstructed. Native FLOW velocity gradients supply strain and vorticity at all published owner nodes. The existing SST-V2003m postprocessor evaluates

$$\mu_t = \max\left(0, \frac{\rho a_1 k}{\max(a_1 \omega, SF_2)}\right), \quad (44)$$

with its native strain floor and blending functions. The saved numerical carry contains recovered modal temperature, this fine-grid eddy viscosity, a zero reconstruction-failure counter, a false first-order flag, and zeroed padding. This creates a fresh reconstruction history for the new initial guess. The fourteen-component state and twenty-four-byte carry are distinct from a conventional SU2 restart file.

All 42,396,569 initialized nodes passed native EOS, species positivity, charge, transport, and energy round-trip checks. The thermal check compares native recovered internal energy with $q_E - |\mathbf{m}|^2/(2\rho) - \rho k$; the modal check compares ρe_v with q_v . Their relative denominators are the larger magnitude of reconstructed and stored energies with a $\rho 10^{-30}$ safeguard. Maximum errors were 2.07543×10^{-14} and 1.43917×10^{-14} , respectively, against the pinned 10^{-8} tolerance. The maximum relative electron/ion molar-density defect was 1.03271×10^{-15} . All 615,936 fine wall nodes received the prescribed kinematic adjustment; 415,197 wall ω values changed. The largest absolute thermal-energy roundoff from wall adjustment was $1.16415 \times 10^{-10} \text{ J m}^{-3}$. These are direct checks of this two-temperature SST initialization; existing admissibility theory for single-temperature reacting discretizations [18] supplies context rather than a theorem for the present coupled transfer.

F. Measured Execution and Interpretation of Acceptance

The final geometric lookup took 105.054 s, including a 1.487 s hierarchy build. The verified state-construction pass took 39.508 s; it reused an exact local clone and compared every resulting chunk with a fresh calculation. That timing therefore describes verified reconstruction with reuse. Full native initialization took 162.581 s

with a measured peak resident set of 13,077,512,192 bytes. These stages perform zero CFD updates. They qualify a recoverable starting state and explicitly preserve the source fields and receiver geometry.

Separately, an exact same-input full-grid comparison advanced fine checkpoint 66 to 67 with the previous and optimized native executables. All fourteen residual norms, restriction counts and scales, native linear norms, final state bytes, and numerical carry bytes were identical. Native verification times were 646.954 and 513.116 s, an observed ratio of 1.261. Background loads differed, with compilation and qualification overlapping the reference run, so this observation is reported with that execution context rather than as an isolated hardware benchmark.

The new transferred branch subsequently completed its first native coupled update in 571.833 s. All 42,396,569 FLOW, SST, thermodynamic, and transport rows were covered with zero rejected final states. Its true global, k , and ω linear residual ratios were 2.73236×10^{-16} , 1.58801×10^{-10} , and 2.73236×10^{-16} . The same update executed 1,935 FLOW GPU replay dispatches and 1,452 fused flux-correction dispatches; the SST linear solve used the qualified CPU path. These are measured software-execution and solved-direction results. The accepted epoch is a completed steady update, and nonlinear convergence, shock-resolution assessment, and comparison with experimental physics remain separate result criteria. Immutable checkpoint retention includes the admitted starting state, accepted epochs 1–3, every epoch index divisible by one hundred, the final requested update, and a completed update carrying a graceful-stop request. Recoverable accepted working state is maintained between these checkpoints.

The first update recorded 27,415,764 canonical first-order selections among 253,509,808 edge-endpoint visits. This count includes native reconstruction-history persistence: a failed endpoint event sets the first-order flag, which clears only after more than twenty consecutive successful events. It measures endpoint visits rather than unique nodes; a later thermodynamic consistency check can also select first order and is not included in this counter. Species restrictions affected 449,236 unique owner rows, and the subsequent thermal search restricted 6,153 owner rows. The minimum species scale was 1.86504×10^{-50} ; the minimum effective local pseudo-time increment was 2.20356×10^{-59} s, compared with nominal increments spanning 9.59561×10^{-17} to 1.00797×10^{-6} s. Thus execution time and nonlinear progress are distinct measured quantities: these local restrictions identify the regions requiring subsequent residual and state-evolution analysis.

IX. Verification and Physical Comparison Framework

A. Transfer Consistency and Admissibility

Coarse-grid initialization of finer solutions is established multilevel practice [13]. The present contribution concerns its implementation for a charged, two-temperature flow coupled to SST on distinct native hexahedral meshes. Geometric containment, state construction, thermodynamic recovery, and subsequent solver advancement are separately testable operations. Nonmatching-interface and conservative-volume-transfer research provides the relevant distinctions between pointwise consistency, geometric agreement, and integral conservation [14, 15, 16].

Let the density-weighted transfer state be

$$\hat{Q} = (\rho_s, m, q_E, q_v, \rho k, \rho \omega)^T, \quad \rho = \sum_{s=0}^6 \rho_s, \quad (45)$$

where q_E includes formation, modal, mean kinetic, and turbulent kinetic energies under the implemented convention. The density-weighted transfer vector $\hat{Q}$ is distinct from the stored compact state Q in Eq. (32), whose final entries are the specific variables k and ω .

In a containing donor hexahedron,

$$\begin{aligned} x &= \sum_{a=1}^8 N_a(\xi) x_a, \\ \hat{Q}^f &= \sum_{a=1}^8 N_a(\xi) \hat{Q}_a^c, \\ N_a(\xi) &= \prod_{j=1}^3 [b_{aj} \xi_j + (1 - b_{aj})(1 - \xi_j)], \quad b_{aj} \in \{0, 1\}. \end{aligned} \quad (46)$$

The inverse map is checked by natural-coordinate containment, forward-position residual, and competing-cell ambiguity; invertibility assumptions require geometric examination [17]. For $\xi \in [0, 1]^3$, nonnegative weights sum to unity. This preserves nonnegative interpolated species densities and linear charge relations. Specific k and ω are recovered by dividing their interpolated density-weighted quantities by ρ .

Thermal admissibility additionally depends on the coupled energy decomposition and native modal inversion. Contemporary reacting-flow positivity analysis motivates this explicit admissibility treatment [18]; its single-temperature discontinuous-Galerkin results do not directly establish a theorem for this two-temperature SST transfer. Unlocated receiver points retain their exact accepted fine-grid state. The receiver coordinates and topology remain fixed, and the fallback set is separately quantified. Native wall, EOS, transport, and SST recovery then establish a usable initial state.

Nodal Q1 interpolation does not impose equality of donor and receiver volume integrals. Conservative Galerkin and polyhedral intersection remaps impose additional integration constraints [15, 16]. Accordingly, this transfer initializes a steady solve. It supplies neither a physical-time history nor a conservative remapping certificate.

B. Verification of the Implemented Equations

Verification addresses the particular residual, source, boundary, and update implementations. A smooth manufactured state U^* defines forcing

$$\begin{aligned} S_{\text{MMS}} &= \partial_t U^* - S(U^*) \\ &\quad + \nabla \cdot [F_c(U^*) - F_v(U^*)]. \end{aligned} \quad (47)$$

Componentwise errors on geometrically controlled refinements give $p_{\text{obs}} = \log(e_h/e_{h/r})/\log r$. Smooth-solution order tests assess the expected discretization order, while shock tests assess nonlinear robustness and resolution. Freno et al. [19] demonstrate thermochemical manufactured solutions and explain why shared source evaluations can conceal algebraic source defects. Independently evaluated reaction, relaxation, and property functions therefore complement spatial-order tests.

For the present fourteen-component system, useful checks include species production, modal exchange, electron pressure work, transport projections, SST production/destruction, and the energy derivatives used by the coupled update. Formation-energy conventions and temperature grouping follow explicitly documented model definitions [21, 12, 22]. A round trip through the same EOS establishes internal consistency; independent formula or reference-code agreement provides an additional check. The benchmark programs described here define further evidence targets; their reproduction is not claimed in this section.

C. Experimental Comparison Hierarchy

An observable is compared only after matching geometry, freestream composition and enthalpy, wall temperature and catalysis, transition specification, and measurement location. The HEG cylinder provides complementary shock-layer density, surface-pressure, and heat-transfer measurements [24]. Maier et al. [4] give a specified SU2-NEMO reproduction with a 45 mm radius, 5956 m/s freestream speed, and a 300 K noncatalytic wall. Pressure constrains compression, whereas heating additionally probes transport, near-wall resolution, and catalytic assumptions.

RAM-C flight measurements supply an independent ionization target [23]. Their probe positions and trajectory conditions are es-

sential to a density-profile comparison. The SU2-NEMO AIR-7 example uses Mutation++ in an inviscid calculation, whereas its turbulent interaction example employs SA [4]. These results establish upstream context; the native AIR-7/SST extension has its own comparison requirements. Electron number density is reported as

$$n_e = \frac{\rho_e}{m_e^{\text{particle}}}, \quad n_{\text{NO}^+} = \frac{\rho_{\text{NO}^+}}{m_{\text{NO}^+}^{\text{particle}}}, \quad (48)$$

in m^{-3} , with charge agreement assessed separately from ion mass fraction. Absolute electron density quantifies the strength of the computed ionization.

Hypersonic SST comparisons should include wall pressure, heat flux, separation, and reattachment under matched incoming boundary-layer conditions [25, 20]. Hoste et al. include two-temperature SST among existing implementations and document sensitivity to stress and energy conventions. The present method therefore identifies its selected SST variant explicitly [6, 7]. Agreement on one observable does not determine the others; each retains its measurement uncertainty and numerical sensitivity.

For a measured observable vector d , the comparison operator samples the numerical solution at the instrument locations and, where required, over its spatial response. With a documented measurement covariance C_d , a useful discrepancy measure is

$$E_d^2 = \frac{1}{N} (g - d)^T C_d^{-1} (g - d). \quad (49)$$

This statistic requires the stated covariance; unavailable correlations must not be silently replaced by independent samples. Numerical grid sensitivity, experimental repeatability, and uncertainty in wall or inflow conditions are reported alongside the observable. Pressure, heat flux, and electron density receive their own scales rather than a pooled error that hides a weakly predicted quantity. For the present evolving solution, successive accepted states also establish whether the observable is still changing. This temporal index denotes steady solver updates; it is not elapsed physical flow time. These procedures specify the analysis to accompany future matched benchmarks, without converting the literature's successful comparisons into results for the present implementation.

D. Shock Geometry and Flux Diagnostics

Pressure isosurfaces at declared levels display three-dimensional compression geometry. Quantitative shock assessment begins with a native-cell gradient normal, oriented from the upstream toward the downstream plateau, and a line that remains within the fluid. Along that line, velocity is recovered from interpolated momentum and species density:

$$u_n = \frac{m \cdot n}{\rho}, \quad M_n = \frac{u_n}{a_f}, \quad (50)$$

where a_f is the implemented frozen nonequilibrium acoustic speed. This definition avoids mixing independently interpolated velocities with density. A stationary compression candidate is assessed using pressure and density rises, plateau variation, and normal-Mach change; a moving front requires velocity relative to its measured propagation speed.

For locally planar, steady, source-free inviscid total balances, the jump relations are

$$\begin{aligned} [\rho u_n] &= 0, & [\rho u_n^2 + p] &= 0, \\ [(q_E + p)u_n] &= 0, & [\rho u_n u_t] &= 0. \end{aligned} \quad (51)$$

Here $[f] = f_2 - f_1$, u_t is tangential velocity, and q_E retains the same formation and turbulent-energy convention on both sides. Constant- γ normal-shock formulas are an additional calorically perfect, fixed-composition specialization; they do not replace these balances for the implemented reacting two-temperature state [21, 12].

A finite sampled region requires the actual viscous and diffusive fluxes. With the positive viscous-flux convention of the im-

plemented equations,

$$\begin{aligned} F_{s,n} &= \rho_s u_n - F_{s,n}, \\ F_{m,n} &= \rho u_n^2 + p - \tau_{nn}, \\ F_{E,n} &= (q_E + p)u_n - F_{E,n}. \end{aligned} \quad (52)$$

The total-energy flux includes the implemented stress work, mode-specific conduction, and species enthalpy transport. The documented V2003m stress omits the isotropic ρk term; the total-energy approximation also omits modeled k diffusion. Diagnostic balances must use these actual choices [6, 7, 20]. Species and modal-energy jumps additionally contain their integrated production and relaxation sources. A finite control volume requires all side-face fluxes, preventing a curved or three-dimensional interaction from being mistaken for a one-dimensional isolated shock.

For a steady control volume V , the corresponding balance defect is

$$\mathcal{R}_V = \int_{\partial V} (F_c - F_v) \cdot n \, dA - \int_V S \, dV. \quad (53)$$

State-reconstructed flux profiles provide physical diagnostics. An exact discrete conservation audit additionally requires the native face fluxes, source quadrature, and ownership assembly used by the solver. Their distinction is retained when reporting sampled jump defects.

The pressure-rise width is defined by plateau-normalized pressure:

$$\theta_p(s) = \frac{p(s) - p_1}{p_2 - p_1}, \quad w_{10-90} = s_{0.9} - s_{0.1}. \quad (54)$$

Plateau windows, crossing-selection rules, sample spacing, and intersected native cells accompany the result. Multiple rises, reflections, or wall interactions are reported as compound compression rather than assigned a unique shock thickness. Flux/limiter and grid studies are essential because resolution and dissipation interact strongly in hypersonic flows [26]; shock stability and heating accuracy also require distinct checks [9].

E. Evidence Attached to the Present Case

The present comparison framework connects implementation tests, admitted states, and native field diagnostics to specific physical observables. Transfer coverage and EOS recovery qualify initialization. Accepted updates document subsequent numerical evolution. Grid and normal-transect analyses quantify compression structure. Matched experimental pressure, heating, and species-density comparisons establish physical agreement when those comparisons are executed. This hierarchy makes each contribution assessable without substituting residual magnitude, visual sharpness, or successful file output for the corresponding physical evidence.

X. Literature-Based Case Definition and Comparison

A. HiFiRE6 Configuration and Operating-Point Traceability

The supplied geometry is treated as a HiFiRE6-family computational configuration. The program history describes the inward-turning inlet and integrated waverider architecture[28]. Bisek's 202E investigation provides the closest directly specified operating point: Mach 6, $T_\infty = 223.472222 \text{ K}$, unit Reynolds number $3.638307 \times 10^6 \text{ m}^{-1}$, and an unpowered, adiabatic-wall flow path[2]. The fine-grid freestream pressure is 2027.81585988 Pa, selected with the implemented native transport properties to reproduce that unit Reynolds number. The donor pressure is 1897.27354577 Pa; its corresponding native unit Reynolds number is approximately $3.40409 \times 10^6 \text{ m}^{-1}$. Both branches retain their specified conditions. The transferred state is consequently a developed initial guess for continuation under the receiver conditions.

The geometry audit compares the published 115-in reference length with the 3.75-in duct diameter. The scale factors inferred independently from those dimensions differ by 40.14%. Accordingly, the adopted 2.921-m reference length records the dimensional convention used in this computation; matching that one dimension does not establish the supplier's geometric revision.

The supplied half-domain contains 20,981,248 cells. Reflection

adds 20,981,248 cells, and joining at $y = 0$ gives the 41,962,496-cell, 42,396,569-unique-node full receiver. The join is an internal mesh interface; the solved full-domain state has no imposed symmetry boundary condition there. All cells of this joined source-resolution receiver remain unchanged during transfer. Its outlet is $x = 4.305532$ m and its outer radius is 0.774047 m; the donor's additional outer and wake regions are excluded. Coordinate directions are also stated explicitly: vertical and symmetry-normal directions in Bisek's figures require transformation to the present vertical z and symmetry-normal y before comparing locations or vector components.

Bisek uses SA-noft2 RANS and separate implicit large-eddy calculations, whereas this work uses the specified SST variant and reacting two-temperature AIR-7. His lifting and control surfaces are removed while the outer mold line is retained to couple nozzle behavior to external pressure. These distinctions make pressure structure, mass capture, separation topology, and adiabatic wall temperature useful qualitative comparison observables. A quantitative validation claim requires a matched geometric and physical case. His time-resolved pressure spectra and RMS quantities require time-accurate sampling; the present numerical iteration history measures steady local pseudo-time advancement.

B. Reference Figures and Experimental Observables

The parent HIFiRE6 flow image reproduced in Mileski's thesis is attributed to Bisek's earlier 202E CFD[3]. Its streamline processing uses a +4-deg parent calculation, whereas the present angle of attack is zero. The surrogate developed in that thesis has a modified rectangular isolator; the reference image therefore carries its original geometry, incidence, and numerical provenance. It supplies a physically useful separation comparison when those differences are stated.

Ramunno et al.[1] motivate integrated aeropropulsion analysis and provide the publication layout reference. Their model uses a shock-free quasi-one-dimensional isolator and linearly interpolated inlet properties, followed by equilibrium combustion and a nonreacting nozzle. Those assumptions support economical system analysis. The resolved pressure surfaces and nonequilibrium species in this paper are evaluated through the spatial CFD equations and their own diagnostic evidence.

Recent three-dimensional inlet experiments sharpen the required observables. Noftz et al.[29] measure inlet startability and wall pressure in a Mach-6 facility, including backpressure forcing and distinct model scales. Bustard et al.[30] combine fast pressure-sensitive paint, temperature correction, and pressure transducers to resolve mean pressure, shock locations, and unsteady interaction in a circular-to-rectangular inlet. Johnson and Narayanaswamy[31] resolve restart dynamics with pressure and schlieren measurements in a different streamtraced inlet. These experiments establish valuable diagnostic practices; geometry, Reynolds number, wall thermal response, incidence, and backpressure must be reproduced before adopting their measured fields as quantitative targets.

The present calculation contains no fuel injection. Air dissociation and ionization are distinguished from combustion. The HIFiRE2 direct-connect investigation of Storch et al.[32] illustrates the additional information needed for a fueled comparison: measured inlet state, injector and cavity geometry, fuel staging, wall conditions, and the resulting combustor-induced shock train. Its combustion efficiency and operability measurements define a future fueled-case validation route rather than a result of this air-only computation.

C. Boundary Fluxes and Heat-Transfer Interpretation

For a declared physical boundary Γ , resolved state fluxes are integrated with the native geometry and outward normals. The mass inventory diagnostic is

$$\dot{m}_\Gamma = \int_\Gamma \rho u \cdot n \, dA, \quad \epsilon_m = \frac{|\sum_\Gamma \dot{m}_\Gamma|}{\max(\dot{m}_{in}, \dot{m}_{out})}. \quad (55)$$

Incoming and outgoing contributions are separated locally before summation, including lateral boundaries. The retained donor results

Table 14 Full-domain nodal extrema at fine iteration 10

Quantity	Minimum	Maximum
p , Pa	35.8879	355904
T , K	50.0259	2658.34
T_{ve} , K	92.2346	2295.05
$ u $, m/s	0	2011.17
n_e , m ⁻³	1.02550×10^{-75}	0.400054

use the historical normalization by incoming mass flow alone; their reported percentages are identified separately from the symmetric normalization above. This state-based quadrature is reported separately from a discrete numerical-face-flux balance. The joined internal/external domain permits nozzle pressure and external-flow coupling to be examined directly, and the outlet's characteristic state is part of its boundary assessment.

Adiabatic, noncatalytic walls impose zero conductive heat flux and zero species penetration. Their temperature is a calculated adiabatic-wall response. Validation against a measured nonzero wall heat flux requires the experiment's wall temperature or thermal-response model to be imposed in a dedicated case. Electron number density and charged number fraction likewise retain their physical units and abundance; logarithmic color variation is interpreted with its measured magnitude.

XI. Preliminary Results on the Fine Receiver Mesh

A. Transferred Initial State and Ten Fine-Grid Updates

The new continuation uses the 42,396,569-node, 41,962,496-hexahedron receiver. Donor iteration 600 supplies an initial guess through the mapping in Sec. 8.4; the additional donor wake and outer-domain cells are excluded. The inverse map locates 41,806,828 receiver nodes, and 589,741 explicitly identified nodes retain their previously accepted fine-grid values before native wall and thermodynamic initialization. This latter set occupies a receiver dual-volume fraction of 9.83912×10^{-6} . The nodal count and volume fraction describe different aspects of coverage, particularly near highly clustered walls.

The receiver retains $p_\infty = 2027.81586$ Pa, whereas the donor used 1897.27355 Pa. Consequently, continuation includes relaxation toward the receiver's specified boundary conditions. Interior pressure is recovered from the mapped conserved state and the native two-temperature model; it is not uniformly reset to the receiver freestream. This distinction is essential when comparing the tenth fine-grid field with the developed donor field. Their difference combines interpolation, fine-wall reinitialization, changed operating pressure, and ten subsequent native updates, rather than isolating discretization error.

Fine iteration 1 denotes the first complete coupled update after zero-update initialization. The present comparison uses accepted iteration 10, at which the continuation was intentionally stopped and its complete state sealed. FLOW, SST, final thermodynamic recovery, and transport each cover all receiver nodes, with zero rejected final states. Table 14 reports a separate, bounded-memory recovery over every accepted fine-grid node. These global extrema are distinct from extrema of the selected diagnostic sections. Each range is measured independently; the extrema of two different fields need not occur at the same location.

The initialized state already contains $p_{\min} = 35.8883$ Pa and $T_{\min} = 50.1046$ K. The tenth update retains comparable low-pressure and cold extrema. Their presence is therefore inherited through the developed initial guess and native recovery, rather than being generated solely during the ten fine-grid updates. A causal diagnosis requires their locations, local update restrictions, geometry, and residual contributions to be compared on the same cells. Native positivity and energy round trips establish admissibility and algebraic consistency; physical assessment of these regions uses the field diagnostics and operating-point evidence.

B. Thermal and Charged-Species Interpretation

The two-temperature plots report translational-rotational and vibrational-electronic temperatures separately. A local difference

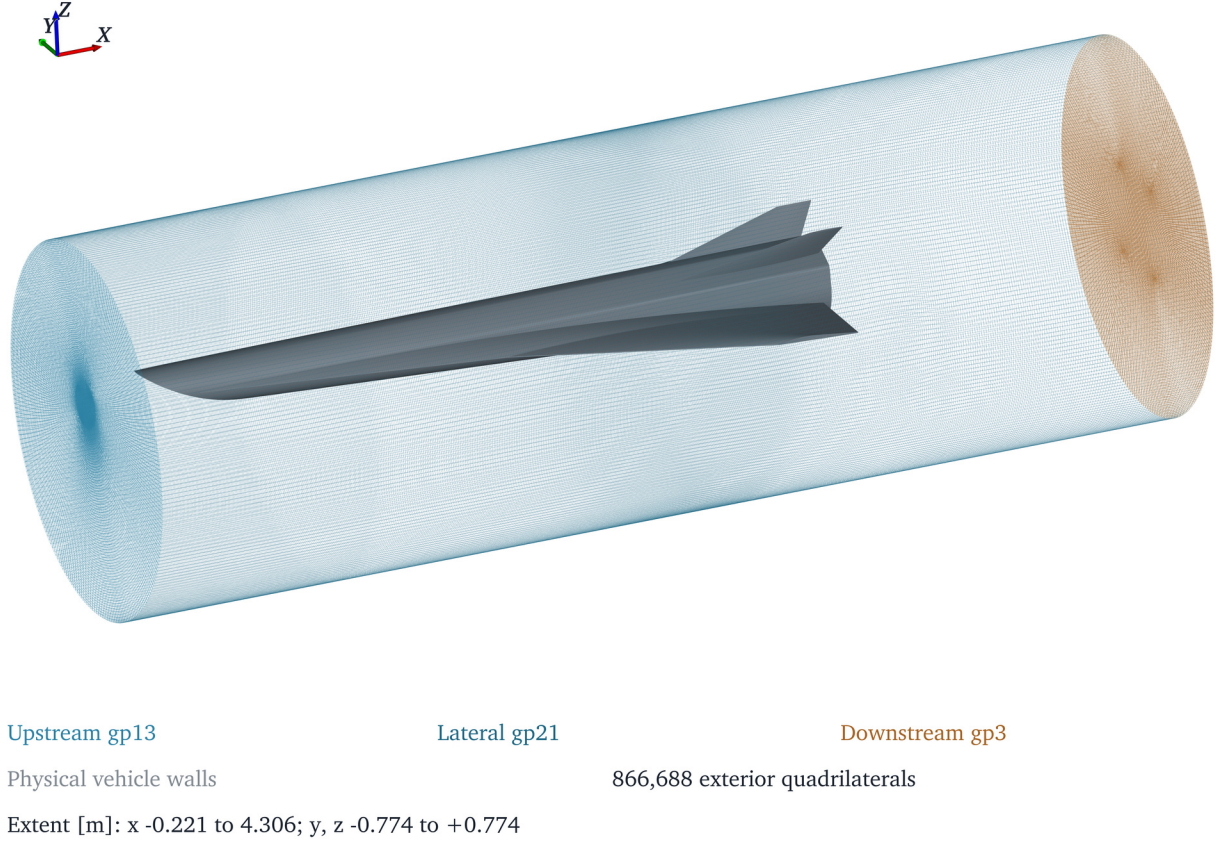


Fig. 6 Fine-grid computational domain and physical boundary zones. The exterior comprises 866,688 native quadrilateral faces around the 41,962,496-hexahedron receiver; 615,936 faces belong to the physical vehicle walls. Upstream gp13, downstream gp3, and lateral gp21 use the admitted far-field condition; the joined midplane and internal engine passages remain interior fluid interfaces. Native geometry is shown in meters without rescaling or an added wake region.

$T_{ve} - T$ is evaluated from colocated values; subtracting the independent global extrema in Table 14 would not measure thermal nonequilibrium. Similarly, the plotted speed is recovered as $|m|/\sum_s \rho_s$, preserving the stored momentum and mixture-density definitions.

The electron number density is calculated with the native molar mass and Avogadro constant, consistently converting the native kg/kmol convention. Its full-domain maximum, 0.400054 m^{-3} , establishes an extremely weak charged population at this iterate. Logarithmic ionization views reveal the spatial distribution of trace species; their color range does not indicate a substantial plasma. The AIR-7 reaction and charge-transport implementation provides the capability to calculate ionization, while the actual abundance remains a measured field quantity. Fuel combustion is absent from this configuration, as specified in Sec. 10.

A plasma-regime screen uses the electron Debye length and electron plasma frequency, expressed in SI units from the standard definitions [33]:

$$\lambda_{De} = \left(\frac{\epsilon_0 k_B T_e}{n_e e^2} \right)^{1/2}, \quad f_{pe} = \frac{1}{2\pi} \left(\frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2}. \quad (56)$$

Using the two-temperature identification $T_e = T_{ve}$ solely for this diagnostic, the independent global bounds give $\lambda_{De} \geq 1047.8 \text{ m}$ and $f_{pe} \leq 5.679 \text{ Hz}$. The lower length bound deliberately combines the minimum mode temperature with the maximum electron density; it is not a claimed colocated extremum. The reference vehicle length is 2.921 m. These formal dilute-species scales support interpreting the images as trace ionization rather than a vehicle-scale plasma sheath. The charge-preserving pair-flux projection remains an implemented transport closure; these scales do not validate a quasineutral sheath or ambipolar regime at vehicle scale. Electromagnetic fields, electron kinetics and radio-frequency propagation

are not solved by this reacting-flow calculation.

C. Matched Three-Dimensional Compression Analysis

Twenty-two fixed-coordinate transects compare accepted fine iteration 10 with donor iteration 600. Two native three-dimensional cell subsets contain the intake and lower-compression probes. Each transect uses 4,001 points at 0.065-mm spacing, conservative momentum/density recovery, independent native-cell containment, and admissible interpolation weights. The same FP64 coordinates query both retained states. Fine-state pressure and sound speed are recovered by the native double-precision thermodynamic implementation. The donor retains its original single-precision pressure and Mach output, with double-precision geometry and conserved variables; its normal Mach uses an approximate sound-speed interpolation.

The local pressure normal is the normalized Q1 pressure gradient in the containing native hexahedron. At the central intake anchor it is approximately (0.66653, 0.64437, 0.37486). The maximum orientation change over twelve declared nearby offsets is 2.3152° at the intake and 1.0900° at the lower anchor. A containing-cell derivative supplies local orientation, rather than a symmetric or surface-averaged shock normal. The projected $y = 0$ line is therefore a distinct geometric transect. Normals are recomputed from iteration 10, and the donor is resampled on those exact lines; the iteration-1 and iteration-10 transects are not asserted to coincide.

The central intake endpoint-to-maximum compression gives $p_{\max}/p_0 = 19.15985$ and $\rho(s_{p_{\max}})/\rho(s_0) = 3.22000$ in the receiver, compared with 19.28071 and 3.24848 in the donor. The corresponding 10–90% compound rise spans 27.8041 mm in the receiver and 28.6791 mm in the donor. Twenty-four and twelve distinct native cells, respectively, are hit by samples in that interval. These sample-

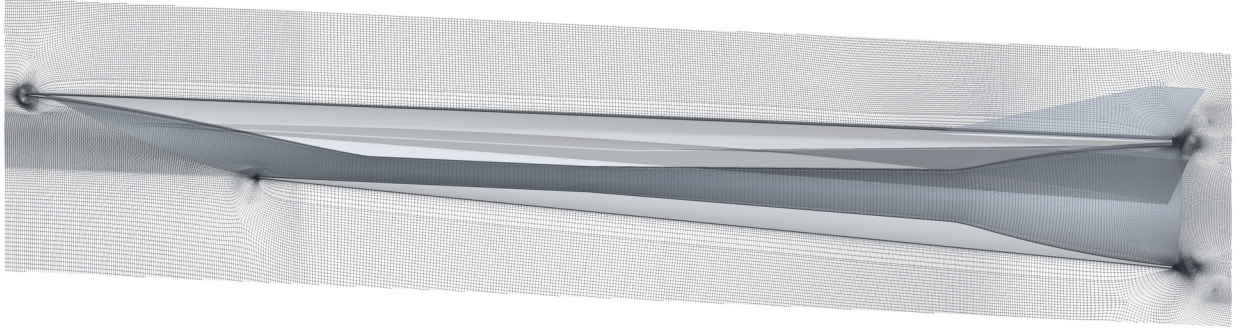
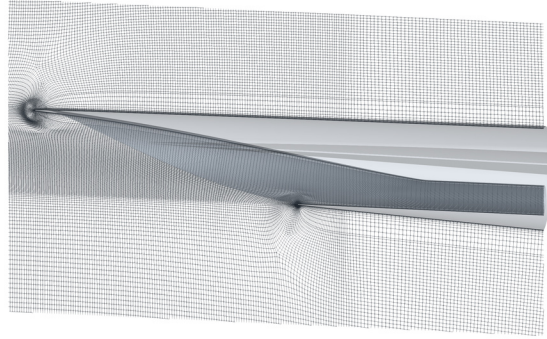
Fine receiver vehicle grid $x = -0.060$ to 3.072 m**Fine receiver intake grid** $x = -0.060$ to 1.133 m

Fig. 7 Fine receiver mesh at vehicle and intake scales, with native cell intersections on the vertical $y = 0$ plane and the physical wall cutaway. The displayed intersections preserve the receiver connectivity and FP64 coordinates. The complete computational domain contains 42,396,569 nodes and 41,962,496 hexahedra.

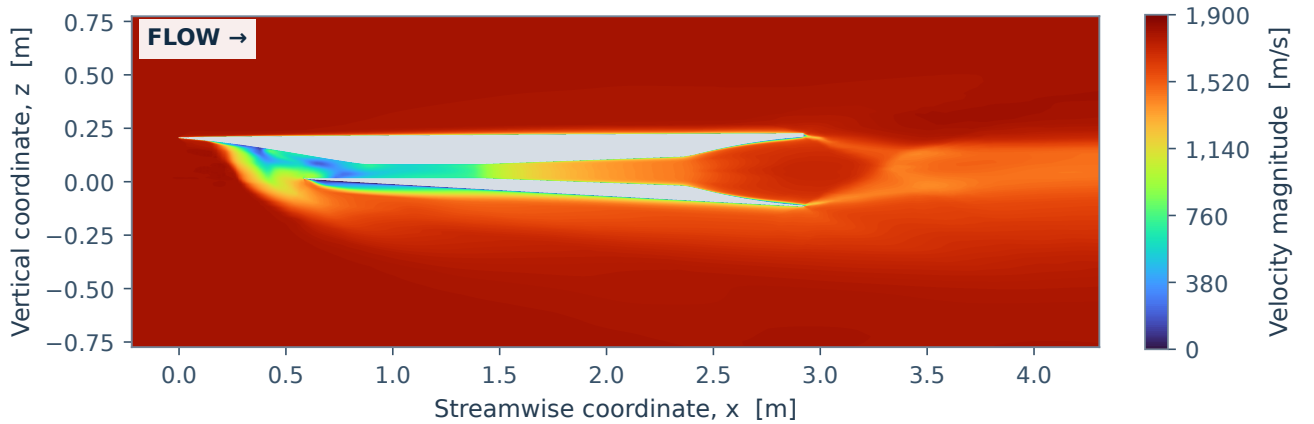


Fig. 8 Early coarse-to-fine continuation at accepted fine iteration 2: velocity magnitude over the complete $y = 0$ fluid section, spanning $-0.22119 \leq x \leq 4.30553$ m and $|z| \leq 0.77405$ m. Speed is recovered from the saved conservative momentum and total species density, with linear color interpolation within 464,654 native cut triangles. The fixed linear display range is 0–1900 m/s; the actual section range is 0–1827.25 m/s. Gray identifies points outside the fluid section. This retained early state belongs to the same continuation branch as iteration 10 and is labeled separately.

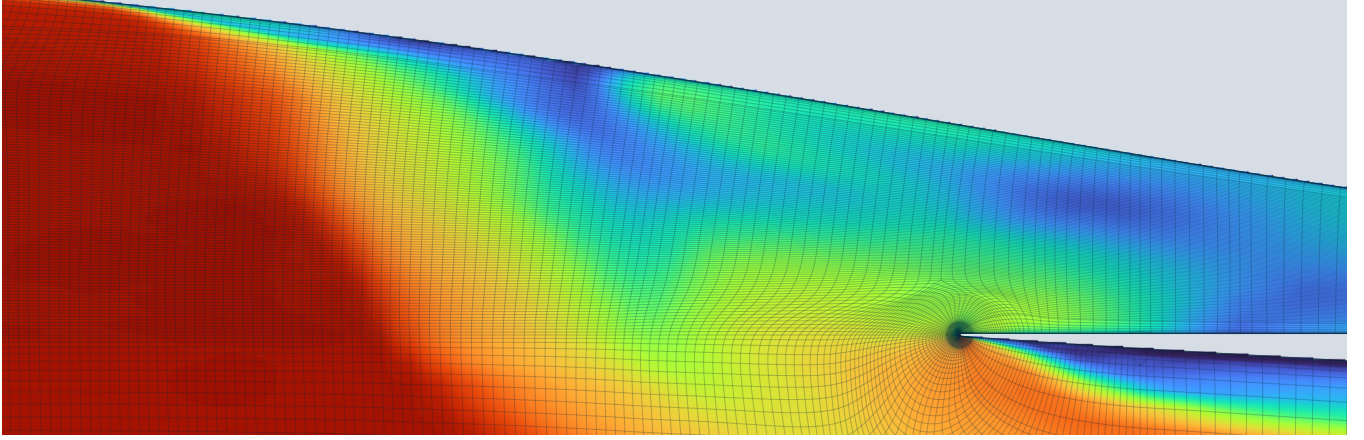


Fig. 9 Native mesh support for the early intake velocity field at accepted fine iteration 3. This magnified $y = 0$ detail retains actual receiver cut-polygon edges over the conservative-velocity magnitude. The complete parent section contains 466,576 native edge segments; the overlay adds no contour-triangulation diagonals. Colors use the same linear 0–1900 m/s range as Fig. 8; the full iteration-3 section maximum is 1820.79 m/s. The earlier overview and this detail represent different accepted iterations, not a matched-state mesh comparison.

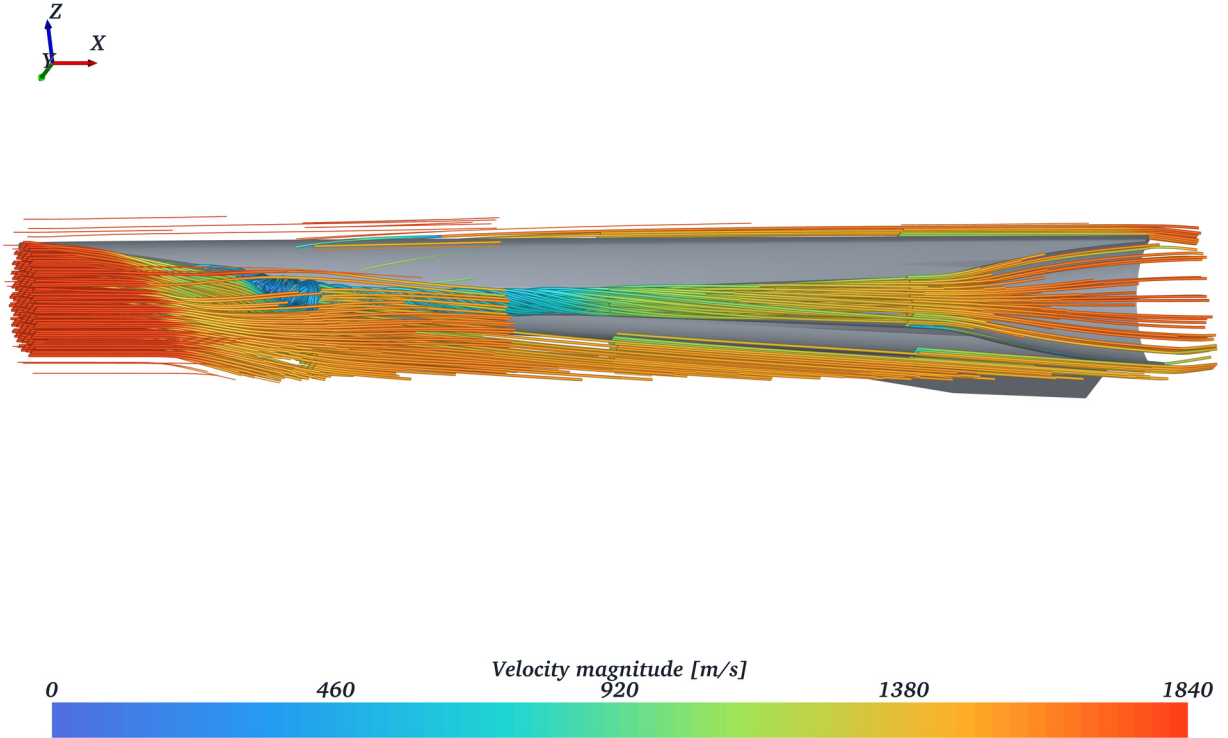


Fig. 10 Whole-vehicle underside context for conservative-velocity streamlines at fine iteration 10. Curves integrate the direction of $u = Q_1(m)/Q_1(\sum_s \rho_s)$ in the exact union of the wider intake and narrow full-length duct volumes, containing 7,482,428 receiver cells. The view contains 463 continuous pieces from 459 selected source trajectories. Every trajectory with a sampled speed at or below 1500 m/s, or entering the declared core $0.18 \leq x \leq 2.95$, $|y| \leq 0.035$, $-0.09 \leq z \leq 0.18$ m, is retained; every eighth remaining fast-exterior trajectory supplies context. Color reports unchanged physical speed. Protected and fast-context tubes use display radii of 3.5 and 1.12 mm, respectively; their cross-sections have no physical meaning.

hit counts are nonexhaustive. The 3.05% smaller sampled width documents this particular transferred-state comparison; changed boundary pressure and finite fine-grid relaxation prevent its interpretation as an asymptotic discretization-order test. The diagnostic endpoint normal Mach decreases from 4.04584 to 0.251411 in the receiver and approximately 4.04551 to 0.252116 in the donor.

Of ten receiver compression candidates, one passes the declared plateau, orientation, and compression-support tests. It lies on the intake family at $y = -20$ mm; the other nine fail downstream-plateau or local-orientation criteria. All nine donor candidates fail the downstream-plateau test. For the receiver candidate, two complete native-cell intervals support each plateau, and pressure, density, and velocity spread and trend each meet the 2% criterion. Its local pressure ratio is 13.0726, density ratio 2.68806, and normal Mach decreases from 3.57170 to 0.295138. Its 26.3857-mm 10–90% numerical rise contains 22 distinct sample-hit cells.

Plateau identification and jump conservation are evaluated separately. With $u_n = u \cdot n$, the inviscid state quantities are

$$J_m = \rho u_n, \quad J_p = \rho u_n^2 + p, \quad H = E + p/\rho, \quad (57)$$

and the reported symmetric difference is

$$d_J = \frac{2|J_d - J_u|}{|J_d| + |J_u|}. \quad (58)$$

For this candidate, $d_{J_m} = 0.68481$, $d_{J_p} = 0.25007$, and $d_H = 0.01830$. Upstream and downstream mass fluxes are 41.4246 and 20.2923 kg/(m² s), respectively. Thus the state has a detected compression front with admissible plateaus and a normal-Mach crossing, but it does not satisfy the stationary, locally one-dimensional inviscid jump balance. These profiles exclude viscous and diffusive fluxes, multidimensional lateral transport, and possible shock motion. They diagnose the saved preliminary state and are not a substitute for the native discrete numerical-flux audit. A single resolved-shock claim would require the relevant conservation and localization evidence in addition to the plateau test.

The lower central transect has a receiver compound pressure ratio of 4.11028 and width 91.1853 mm, compared with 4.11586 and 92.9453 mm in the donor. Fifteen receiver and nine donor cells are hit within these respective rise intervals. Spanwise offsets and $\pm 5^\circ$ direction perturbations expose the dependence of detected cores on three-dimensional structure. Incomplete or overlapping VTK traversal intervals are retained as algorithmic reporting diagnostics; independent native-cell evaluation governs sample validity. They are not interpreted as physical mesh holes or overlapping mesh cells.

D. Localized Intake Evolution

The common intake window $0.10 \leq x \leq 0.23$, $0.02 \leq z \leq 0.15$ m on $y = 0$ isolates the cold, high-Mach region previously observed in the donor. At iteration 10, the native-section minimum pressure is 1517.98 Pa, minimum translational temperature is 209.360 K, and maximum Mach number is 6.24720. The donor values within the same geometric window are 536.955 Pa, 61.6348 K, and 11.9002, respectively. Separate native sections retain each mesh's topology; their extrema need not occur at identical coordinates. The corresponding iteration-1 receiver values were 557.445 Pa, 63.999 K, and 11.6729. This window therefore shows substantial relaxation of the inherited localized extreme over the ten-update continuation. The global cold extrema in Table 14 persist elsewhere. The comparison establishes measured spatially localized evolution, while a causal attribution among wall recovery, reconstruction, and boundary adjustment requires their colocated numerical contributions.

E. Residual Evolution and Local Update Restrictions

The ten accepted updates provide an initial progression record. For equation j , write the reported native assembly norm and its relative progression as

$$r_j^{(n)} = \left[\frac{1}{N} \sum_{i=1}^N (R_{i,j}^{(n),\text{assembly}})^2 \right]^{1/2}, \quad q_j = \frac{r_j^{(10)}}{r_j^{(1)}}. \quad (59)$$

Table 15 Native assembly progression from fine iteration 1 to 10

Equation	q_j	Equation	q_j
ρ_e	2.59437	ρu	0.718386
ρ_{N_2}	0.856801	ρv	0.551041
ρ_{O_2}	0.856801	ρw	0.793444
ρ_{NO}	1.02048	ρE	0.533404
ρ_N	0.169047	ρE_{ve}	0.632511
ρ_O	1.37017	k	1.02541
ρ_{NO^+}	2.59437	ω	3.04051

The FLOW and SST residuals come from their respective native assembly phases. They are not a fresh simultaneous evaluation of all fourteen equations at the final accepted state. Ratios are formed separately for each equation, avoiding a comparison between dimensional residual magnitudes of different equations.

Momentum and both energy norms decrease over this short sequence, while several species norms and the SST norms increase. The tiny absolute charged-species residuals reflect the trace species scales and are reported alongside their relative increase. This is a mixed initial relaxation record, not established nonlinear convergence. The independently verified SST true linear-direction norms pass the declared 10^{-7} threshold for every update; these quantify the remaining linear-system defect of the computed correction within each coupled update.

Species-limited owner rows decrease from 449,236 at iteration 1 to 20,728 at iteration 10; thermal-limited rows change from 6,153 to 5,618. Canonical first-order reconstruction selections increase from 27,415,764 to 44,619,930 out of $2E = 253,509,808$ edge-endpoint visits. This counter includes reconstruction-history persistence and does not count unique nodes. At fixed nominal Courant number 0.5, the minimum effective local increment rises from 2.20356×10^{-59} to 9.59561×10^{-17} s, while the maximum remains approximately 1.0079×10^{-6} s. These are spatially local pseudo-time increments, not a global physical timestep or elapsed flow-development time. Complete diagnostic counters are available at iterations 1–4 and 10; the plotted residual history includes all ten accepted updates. Missing intermediate counter records are not interpolated.

Native execution times range from 568.119 to 630.887 s per accepted update; iteration 10 takes 581.755 s. These timings include the recorded concurrent workstation workloads. The complete ten-update record shows a 46.7% decrease in the total-energy assembly norm and a 36.7% decrease in the mode-energy norm. The ω assembly norm increases by a factor of 3.04, and the k norm increases by 2.54%. This mixed relaxation establishes neither nonlinear convergence nor an asymptotic time-to-solution rate. At iteration 10 the actual true double-precision global, k , and ω linear-direction ratios are 2.5832×10^{-16} , 5.2630×10^{-8} , and 2.5832×10^{-16} , respectively, all below the declared 10^{-7} acceptance threshold. Linear-direction acceptance is evaluated separately from the nonlinear assembly norms. The iteration-10 record demonstrates the role of equation-wise checking: an initial global ratio of 1.58356×10^{-8} already satisfied the threshold, while the k ratio remained 1.86468×10^{-2} . Six additional CPU double-precision Krylov correction iterations reduced the k ratio to 5.26297×10^{-8} , using ten of the twenty allowed linear iterations. The global and component checks therefore evaluate distinct scale sensitivities in the coupled turbulence system.

The run was intentionally stopped after acceptance of iteration 10. Its complete state, numerical history carry, native result, actual linear-gate record, and source binding are retained. This finite continuation establishes a reproducible comparison point for the transfer and execution method, with further physical assessment governed by field evolution, flux balance, local restrictions, and operating-point applicability.

XII. Conclusions

The implemented continuation combines a developed AIR-7, two-temperature, SST donor state with the fine-grid computational domain and a globally ordered, bounded-memory native execution path. The measured result is a recoverable 41,962,496-cell initial state followed by accepted coupled fine-grid updates. The method

retains the complete receiver geometry, identifies nonmatching donor coverage explicitly, transfers the conserved thermochemical variables and density-weighted turbulence variables, and applies the actual fine-grid wall, thermodynamic, and transport routines before advancement.

Geometric mapping, initialization, native execution, and physical interpretation have distinct evidence. The final mapping locates 41,806,828 nodes; the remaining 589,741 use their exact previously accepted fine-grid values before the prescribed native reinitialization. Every receiver node passes the native recovery checks, with maximum thermal and modal energy roundtrip errors of 2.08×10^{-14} and 1.44×10^{-14} . This provides a numerically consistent initial guess across the documented geometric mismatch. Global volume-integral conservation is not imposed by the nodal Q1 transfer.

Ten accepted fine-grid updates preserve the donor's developed compression and thermal structure while producing measurable local relaxation. The central intake compound rise spans 27.804 mm across 24 sample-hit cells, compared with 28.679 mm across 12 donor cells on the same coordinates. Its pressure ratio is 19.160 and diagnostic normal Mach changes from 4.046 to 0.251. One offset front passes the declared plateau, orientation, and compression criteria; its symmetric mass-flux difference of 0.685 separately shows that a stationary isolated inviscid shock balance is not established. The diagnostic framework therefore distinguishes geometrical compression, local plateau evidence, and conservation qualification.

Within the specified intake window, the minimum translational temperature rises from 63,999 K at the first fine update to 209,360 K at the tenth, while the maximum Mach number decreases from 11.6729 to 6.24720. This is localized field evolution rather than a claim that every inherited numerical extreme has disappeared. The full-domain maximum electron number density is 0.400054 m^{-3} ; the ionization figures resolve trace charged-species abundance and do not imply a substantial plasma sheath.

The total-energy and mode-energy assembly norms decrease by 46.7% and 36.7%, respectively, over the ten-update record. The ω norm increases by a factor of 3.04. Native update times of 568.119–630.887 s establish measured execution throughput; the mixed non-linear relaxation does not establish a converged time-to-solution rate. The complete accepted state is preserved at the owner-selected termination point.

The contribution is the implemented and qualified combination of complete-state transfer, explicit geometric disagreement, native recovery, exact computational reuse, and full-domain continuation on a bounded-memory workstation. Coarse-to-fine initialization follows established multilevel practice. The distinctive evidence is the integrated thermochemical and turbulence state contract, its source-level implementation and numerical equivalence tests, and physically discriminating diagnostics at the stated grid scale. Continued validation requires matched operating conditions, sustained nonlinear and observable convergence, and comparisons with applicable experimental or benchmark data.

Data and Code Availability

The numerical record retains source and build identities, actual configurations, complete native fields and restarts, histories, geometric certificates, and measured resource observations. Companion evidence and bibliography ledgers bind quantitative results to their source records. The author maintains these materials with the manuscript and will specify archival access and release terms for submission.

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The author used generative artificial-intelligence tools for assistance with source-code development, literature discovery,

manuscript editing, and internal adversarial review. Numerical fields were computed by the documented native solver; figures derive from the retained numerical states. The author is responsible for the implementation, source verification, interpretations, and manuscript. Internal red and blue critiques constitute author-directed quality review.

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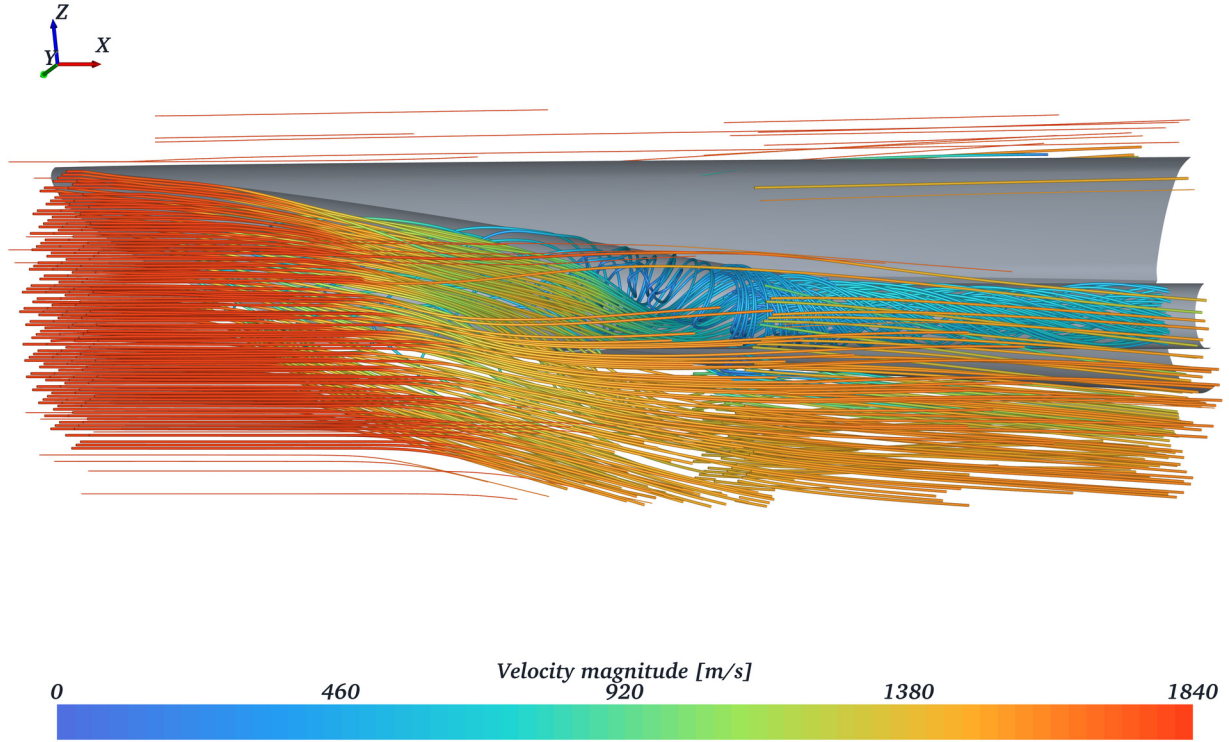


Fig. 11 Broad inlet and duct flow viewed obliquely from below at fine iteration 10, using the same saved conservative-velocity trajectories as Fig. 10. The view retains 363 continuous pieces over $-0.05 \leq x \leq 1.20$ m under the same source-trajectory selection, using 1.4-mm protected and 0.448-mm fast-context display radii. Seven segments and two vertices failed the finite native-cell membership audit and were omitted; for 32 paired seeds, step halving changed matched-arc-length positions by at most $12.743 \mu\text{m}$. Trajectories terminate at the retained fluid extent, including artificial extraction boundaries; they have no elapsed physical-time interpretation.

Full vehicle oblique

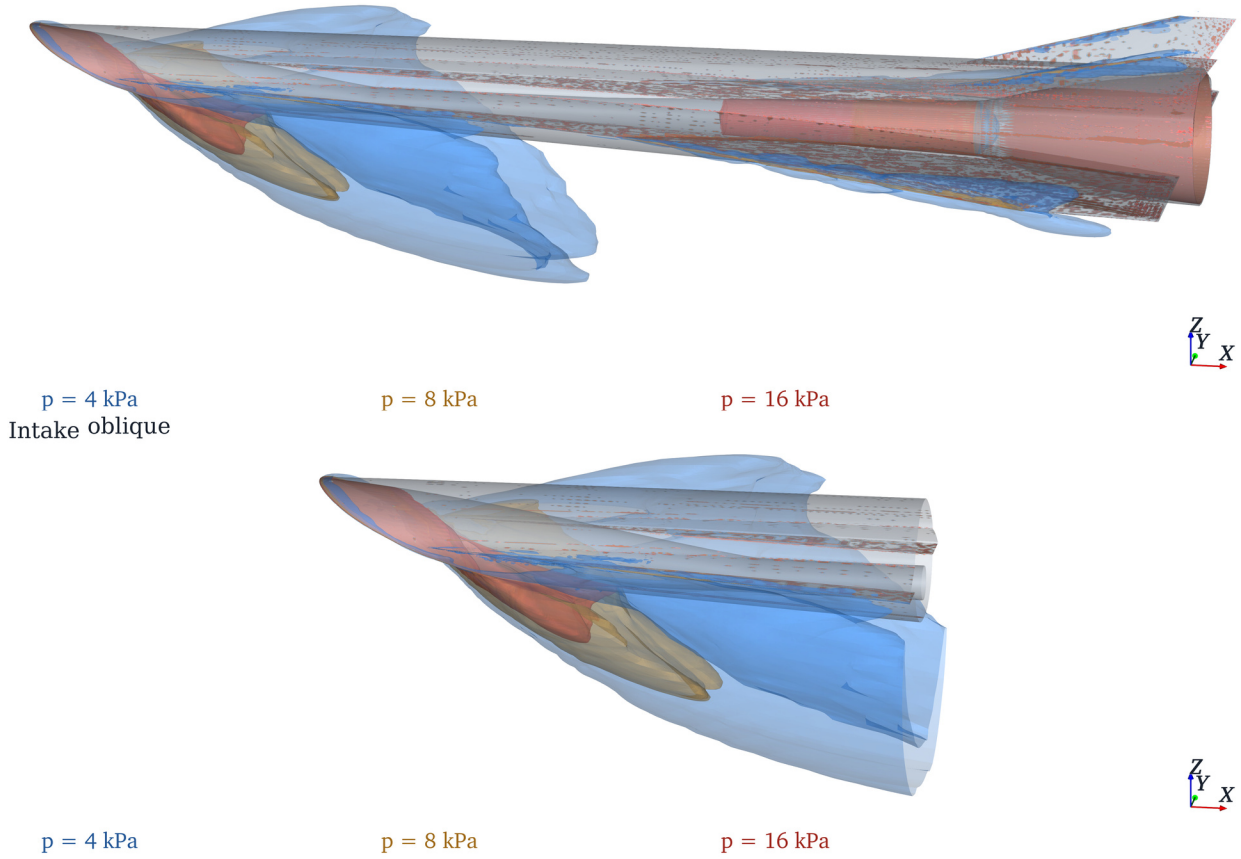


Fig. 12 Three-dimensional compression at fine iteration 10: pressure levels of 4, 8, and 16 kPa in whole-vehicle and intake views. The contour search examined every receiver hexahedron and retained native edge-linear intersections without smoothing, decimation, or added closing surfaces. The physical wall provides context. Pressure levels reveal the organization of compression; local shock assessment additionally uses the matched normal profiles and conservation diagnostics.

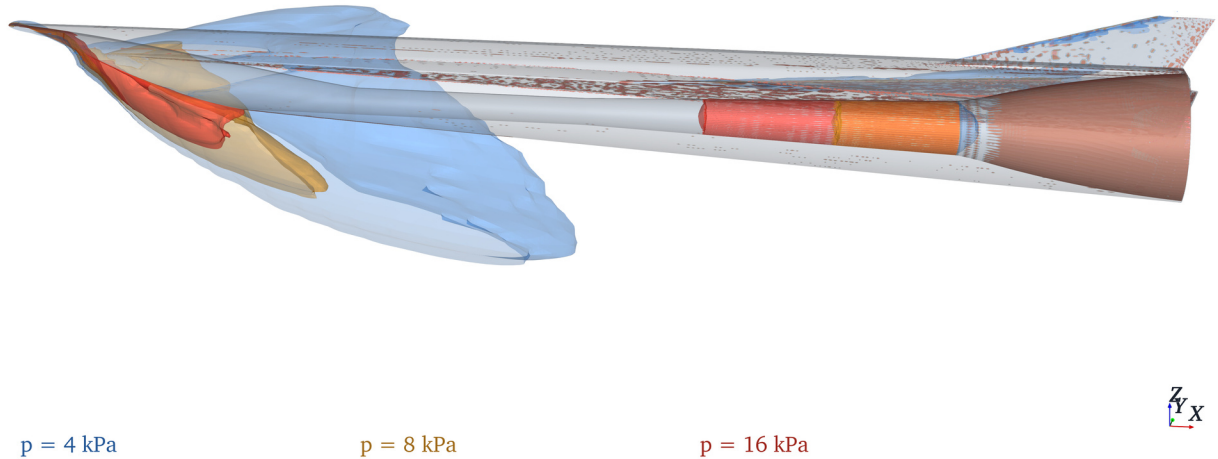


Fig. 13 Whole-vehicle pressure structure exposed at the vertical geometric midplane for fine iteration 10. The $y \geq 0$ portions of the actual vehicle and the 4-, 8-, and 16-kPa isosurfaces reveal the inlet, duct, and rear compression structure. Blue, amber, and red denote the three pressure levels, respectively. Clipping retains open native contour polygons without adding a cap at the cut; the view contains no horizontal sampling plane. These constant-pressure surfaces locate compression structure, with local shock qualification supplied by the independent normal-profile analysis.

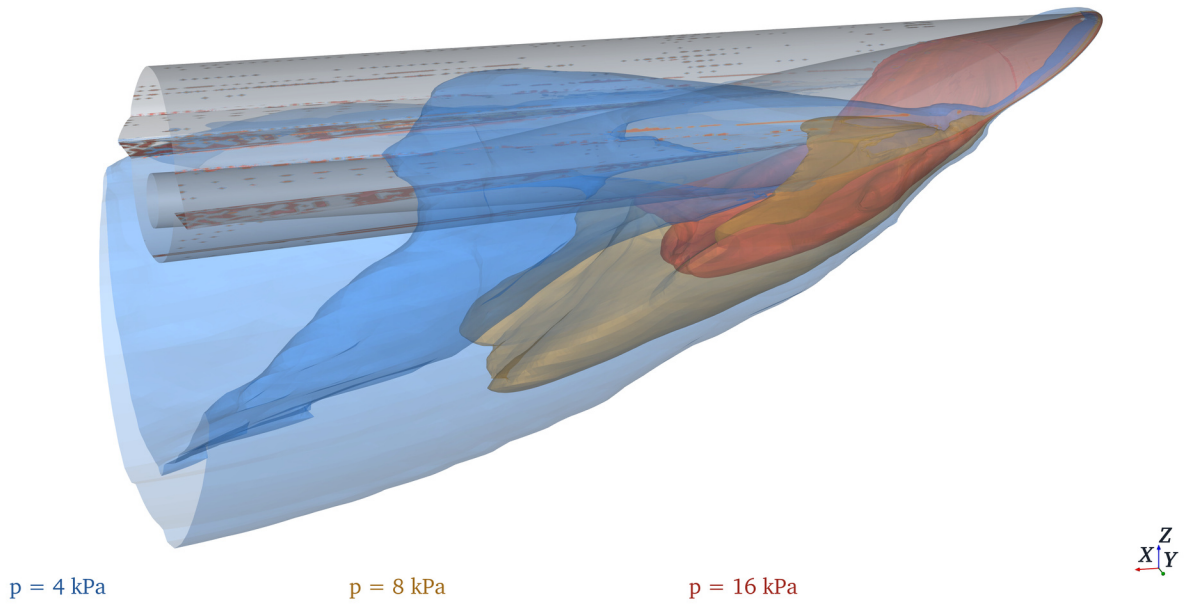


Fig. 14 Opposite-side oblique inlet view of the same fine-iteration-10 pressure surfaces, retaining the actual geometry over $-0.07456 \leq x \leq 1.19303$ m. Blue, amber, and red identify 4, 8, and 16 kPa; transparent gray walls expose the spatial relationship between external compression and the intake passage. The streamwise display crop introduces no closing faces, smoothing, or resampled section plane. This viewpoint supplements the full-vehicle midplane view without changing its field or pressure levels.

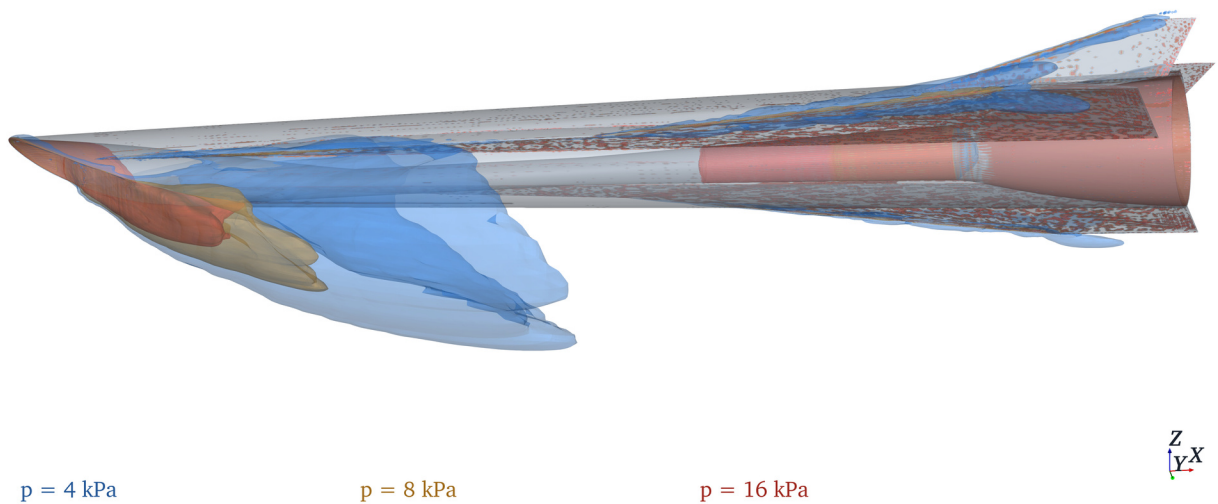
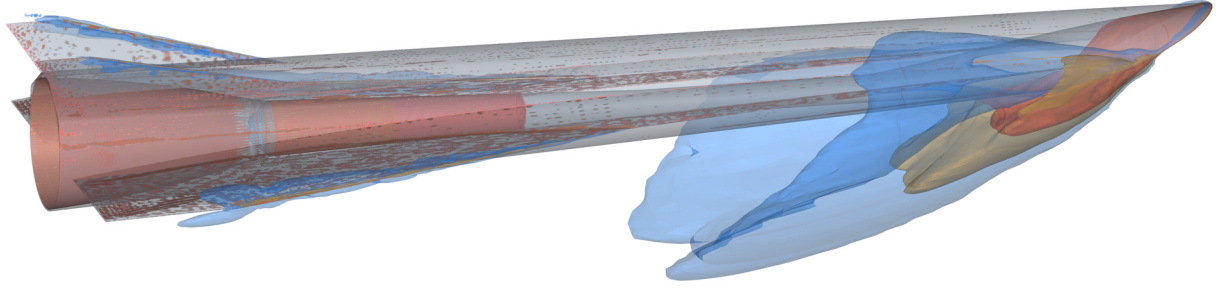


Fig. 15 Whole-vehicle lower-oblique view of the 4-, 8-, and 16-kPa pressure isosurfaces at fine iteration 10. The view shows the lower inlet compression surfaces, their relation to the transparent physical vehicle, and pressure structure along the duct and rear body. Blue, amber, and red retain the same pressure levels as the other compression views. The camera changes while the retained full-domain field, native contour geometry, and physical coordinate scale remain unchanged.



$p = 4 \text{ kPa}$

$p = 8 \text{ kPa}$

$p = 16 \text{ kPa}$

z
 x y

Fig. 16 Whole-vehicle pressure structure from the opposite oblique viewpoint at fine iteration 10. The same blue 4-kPa, amber 8-kPa, and red 16-kPa surfaces expose the spanwise extent of inlet compression and its geometric relationship to the duct, fins, and rear body. Transparent physical walls supply context. This is a second camera view of the identical accepted pressure field, rather than an additional solver state or an independent shock qualification.

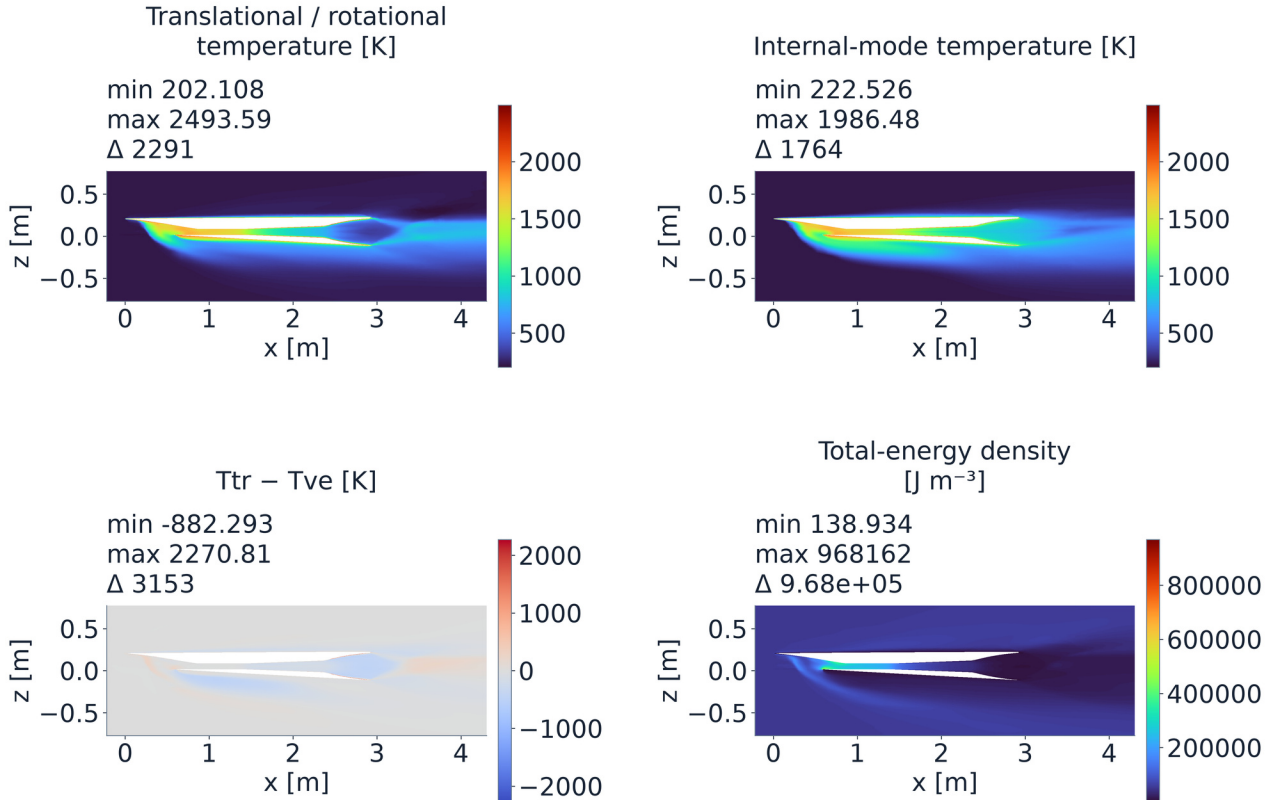
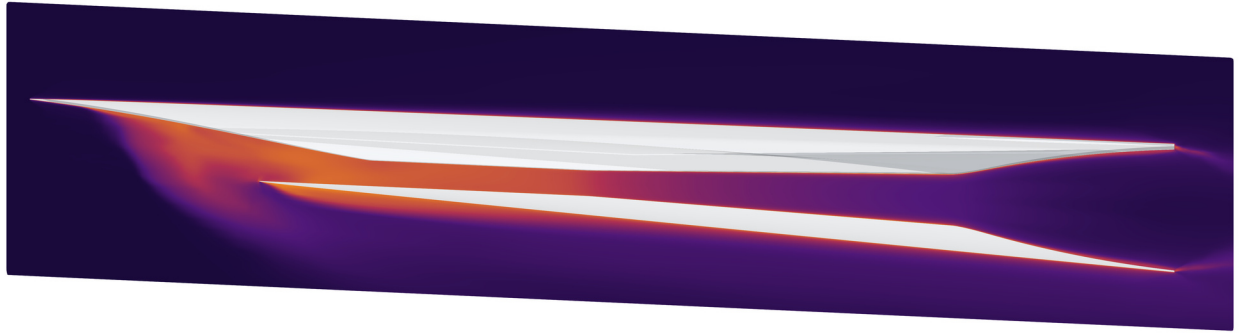


Fig. 17 Translational-rotational and vibrational-electronic thermal fields on the receiver center-plane fluid section at fine iteration 10. Native recovery supplies the two temperatures and conserved energy quantities, with individually declared units and scales. The signed modal temperature difference is evaluated at colocated samples; independent field extrema do not define a local temperature split.

Translation / rotation temperature [K]

$x = -0.060$ to 3.072 m

$\frac{4}{X}$



Vibrational / electronic temperature [K]

$x = -0.060$ to 3.072 m

$\frac{4}{X}$

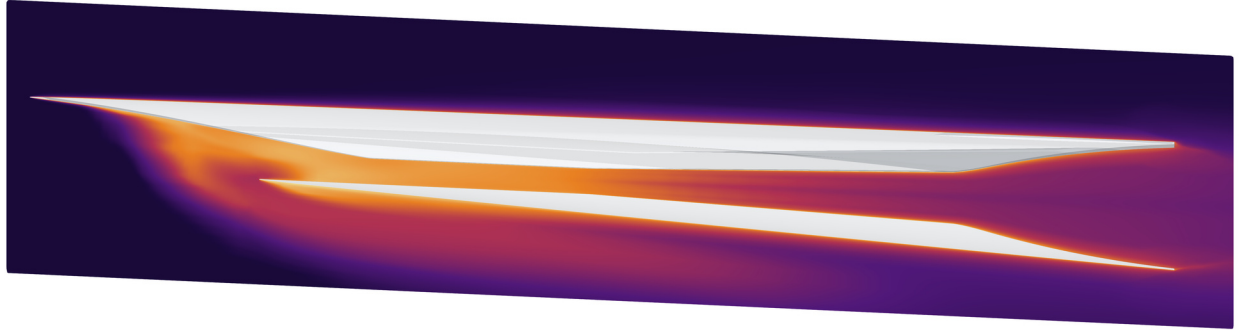


Fig. 18 Oblique vehicle-scale views of the native vertical $y = 0$ thermal section at fine iteration 10, with the physical wall cutaway. The upper panel reports translational–rotational temperature, spanning 209.36–2493.59 K within this display window; the lower panel reports vibrational–electronic temperature, spanning 222.53–1986.48 K. Native two-temperature recovery supplies the nodal values, which are interpolated within the receiver fluid cells. The two linear Kelvin color scales are independent. These section views complement the three-dimensional 800-K isosurface in Fig. 19.

Heated flow oblique

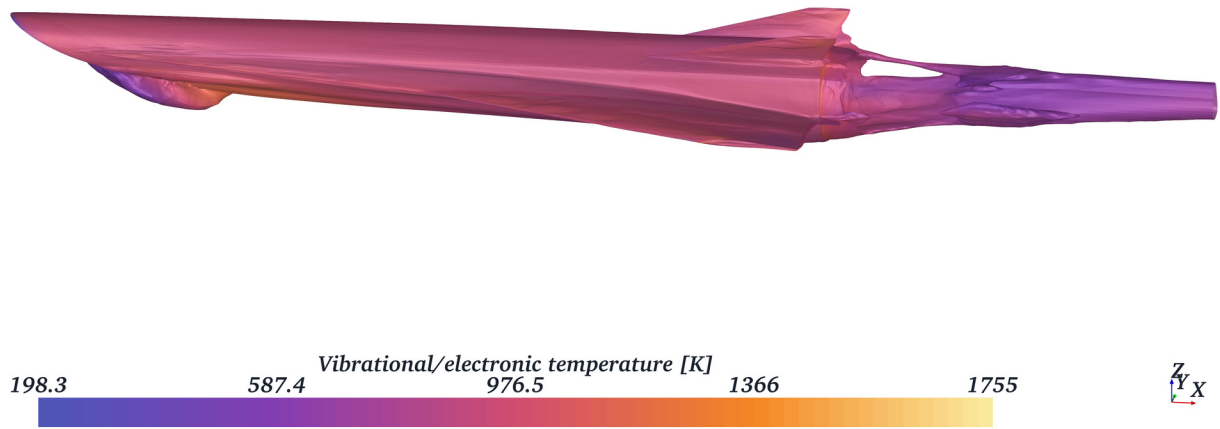


Fig. 19 Full-domain 800-K translational-rotational isosurface colored by vibrational-electronic temperature at fine iteration 10. The mode temperature spans 198.33–1754.63 K on this surface, revealing the three-dimensional relation between translational heating and internal-mode excitation. All 41,962,496 receiver hexahedra entered the native edge-linear contour search. Transparent physical walls reveal the intake and duct context; the displayed surface is an isotherm, not a shock locator by itself.

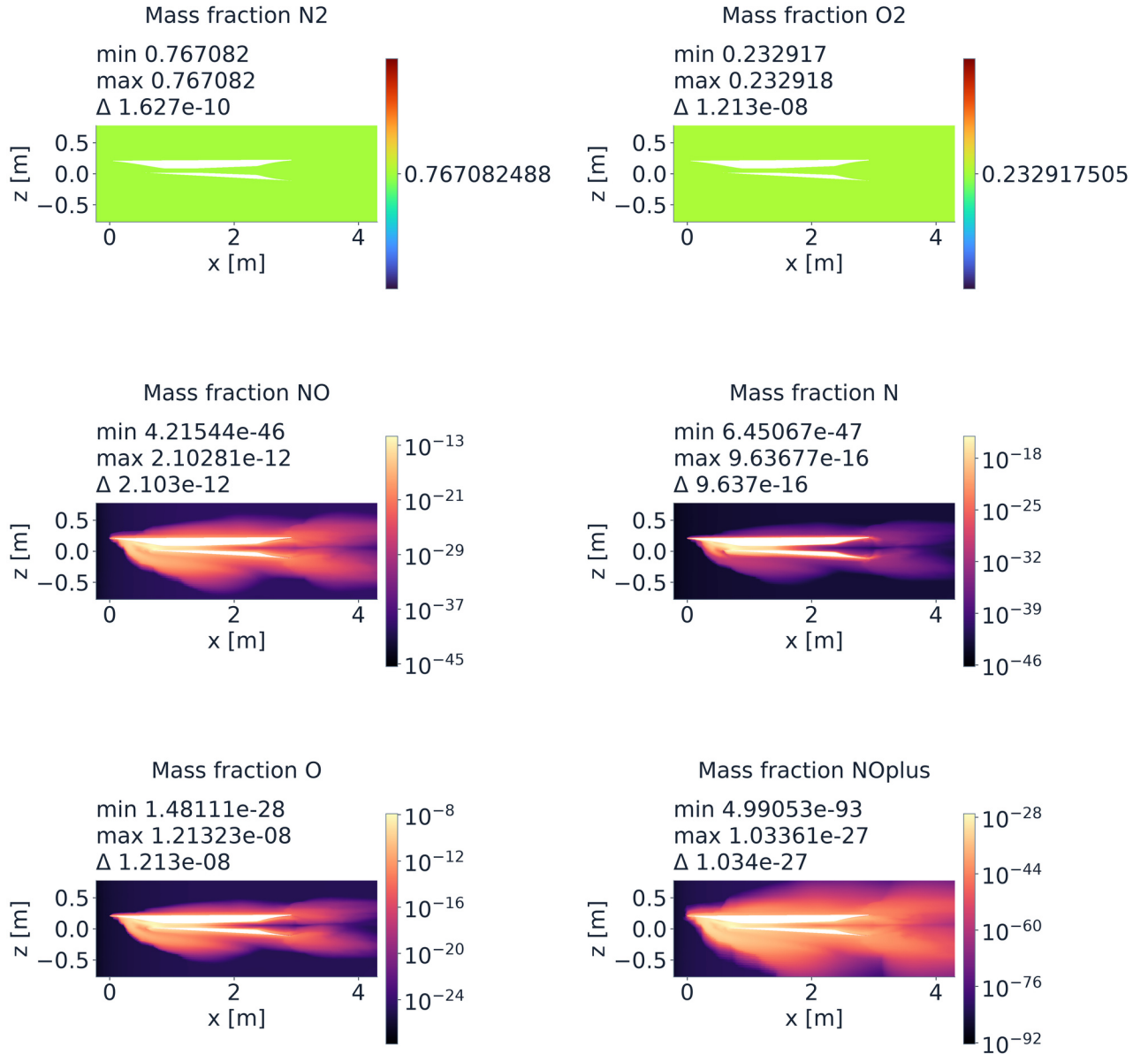


Fig. 20 AIR-7 molecular, atomic, and ionic mass fractions on the fine receiver $y = 0$ section at iteration 10. Each species mass fraction is the ratio of interpolated conserved species density to interpolated total density. Individual colorbars distinguish linear and logarithmic scales; gray denotes exact zero on logarithmic panels. The fields describe abundance in this preliminary fuel-free reacting-air state.

Fine receiver surface: body, wings and fins

$x = 0.000$ to 2.983 m

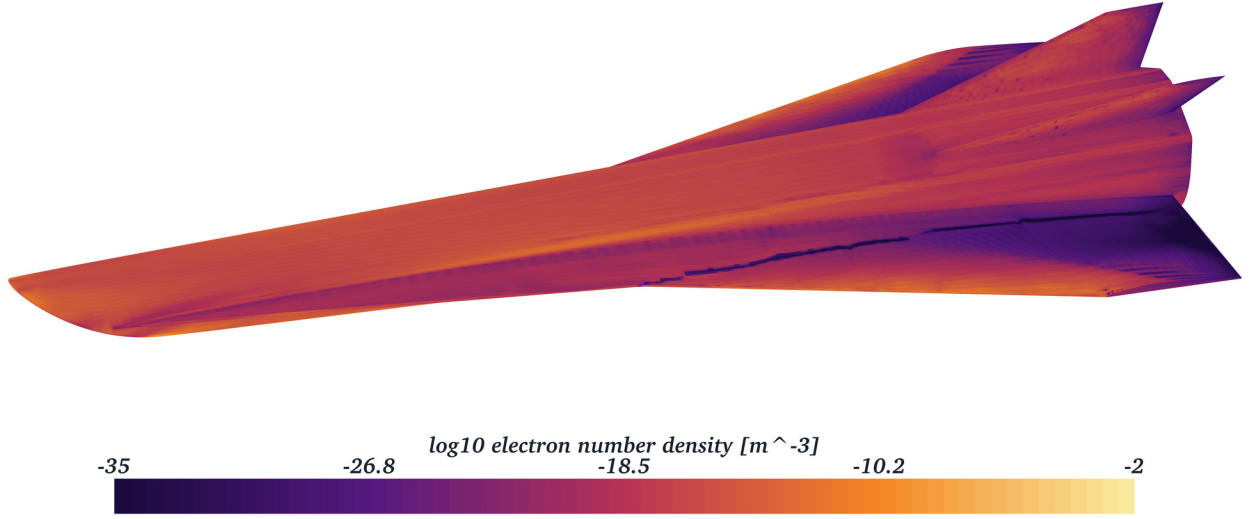


Fig. 21 Trace electron number density on the complete physical vehicle surface at fine iteration 10. Each face color is the arithmetic mean of its four native nodal values, with logarithmic display limits declared on the colorbar. The largest wall nodal value is $3.9943 \times 10^{-3} \text{ m}^{-3}$ and the largest face mean is $2.8833 \times 10^{-3} \text{ m}^{-3}$. The oblique view shows the spatial distribution of a dilute charged population, not a substantial vehicle-scale plasma sheath.

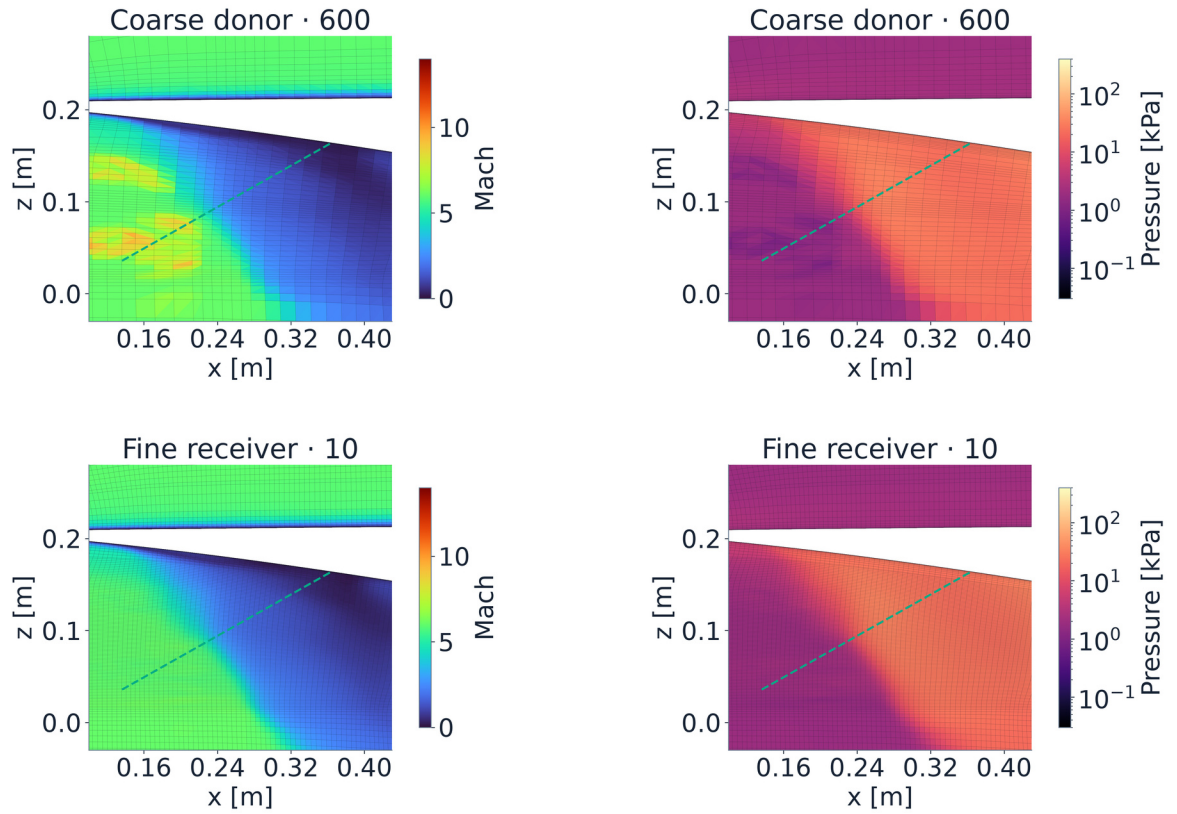


Fig. 22 Matched intake pressure structure and native mesh support for donor iteration 600 and fine receiver iteration 10. Each section retains its own native connectivity in the same physical window. The common three-dimensional probe family supplies the independent quantitative comparison; colored section structure alone does not establish a resolved shock jump.

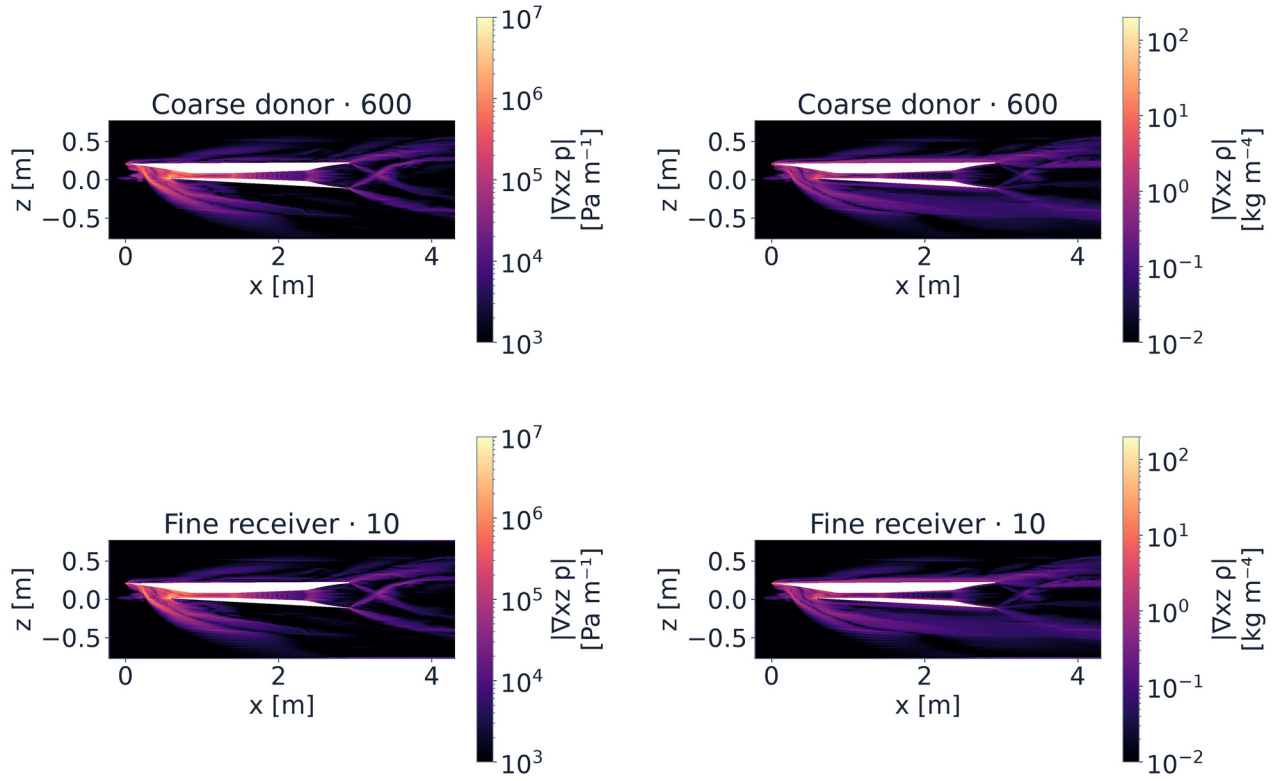


Fig. 23 Center-plane compression footprints in donor iteration 600 and fine receiver iteration 10. Rows compare the two states; columns show the magnitudes of the in-plane pressure and density gradients, $|\nabla_{xz} p|$ and $|\nabla_{xz} \rho|$, on the same receiver x, z extent. Each state retains its independent native section topology. Gradient maps locate compression structure; zeros and values below the displayed logarithmic range use the lowest color.

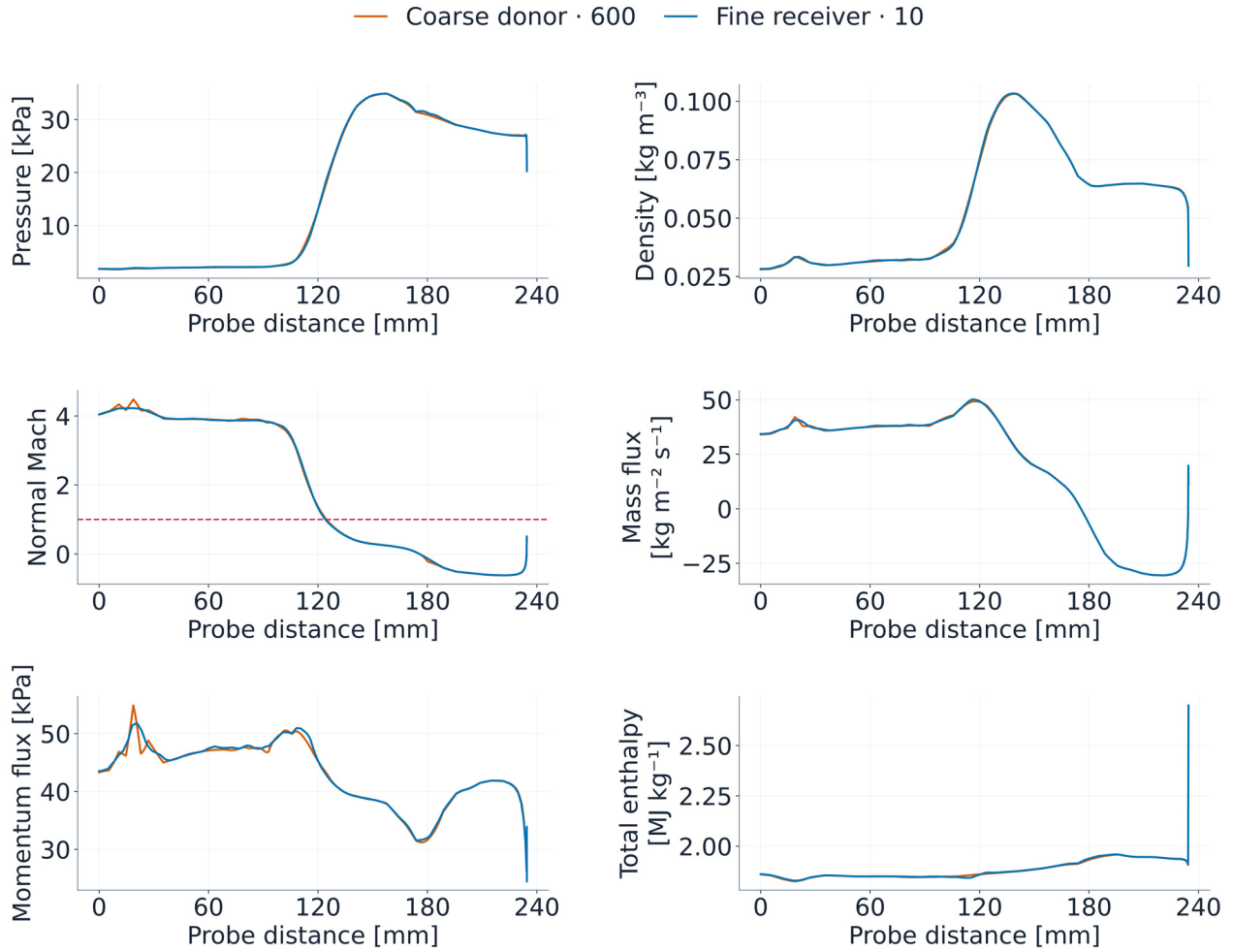


Fig. 24 Matched-coordinate central intake profiles for donor iteration 600 and fine receiver iteration 10. The receiver compound pressure ratio is 19.15985 and the 10–90% rise width is 27.8041 mm, compared with 19.28071 and 28.6791 mm in the donor. Twenty-four and twelve distinct native cells, respectively, are hit within the sampled rise interval. These compound compression measures and normal-Mach crossing are separate from the plateau and jump-balance tests described in the text.

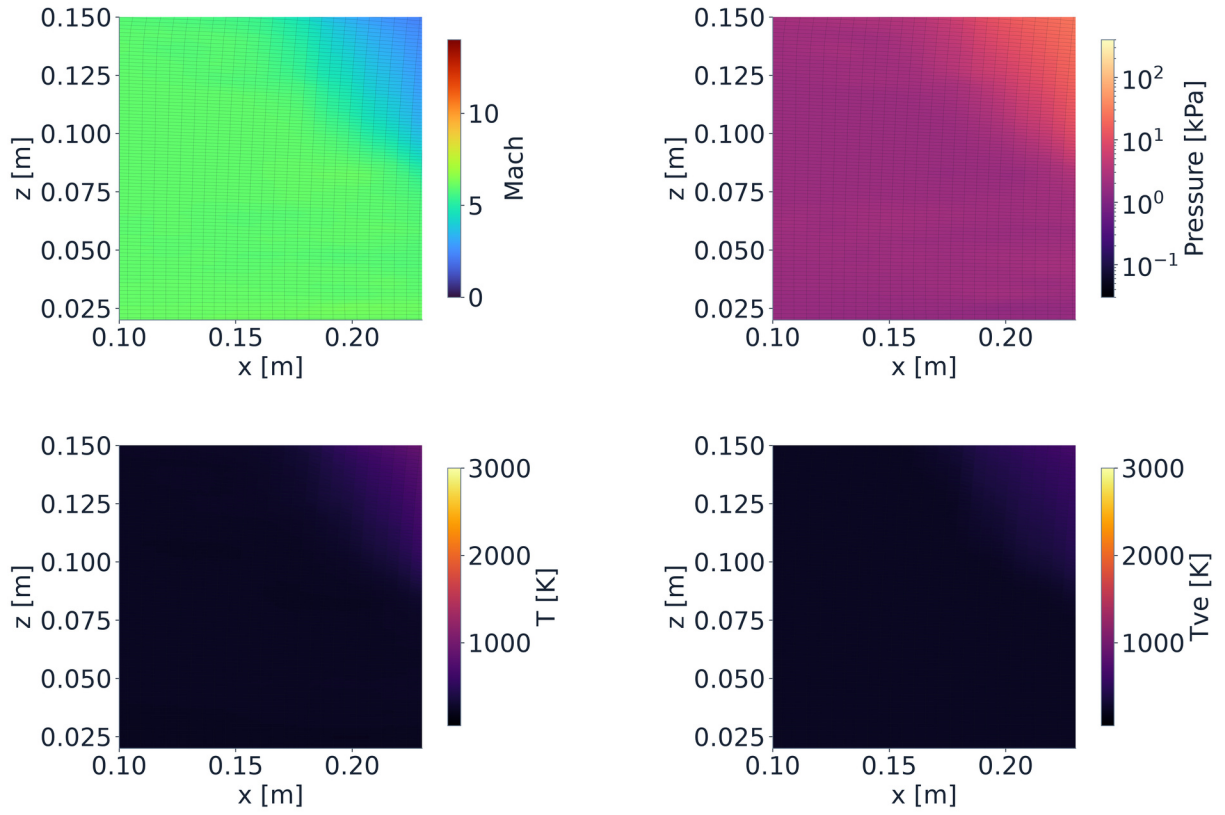
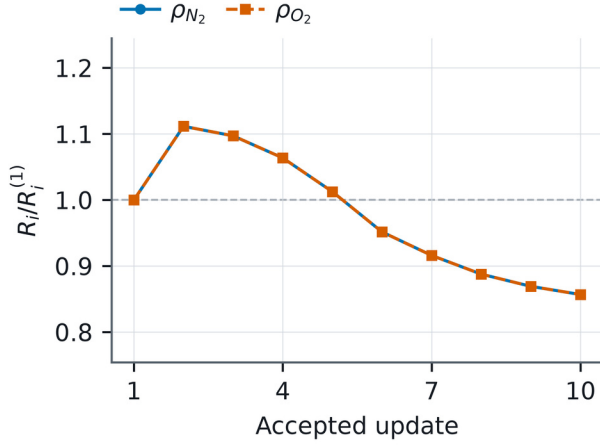
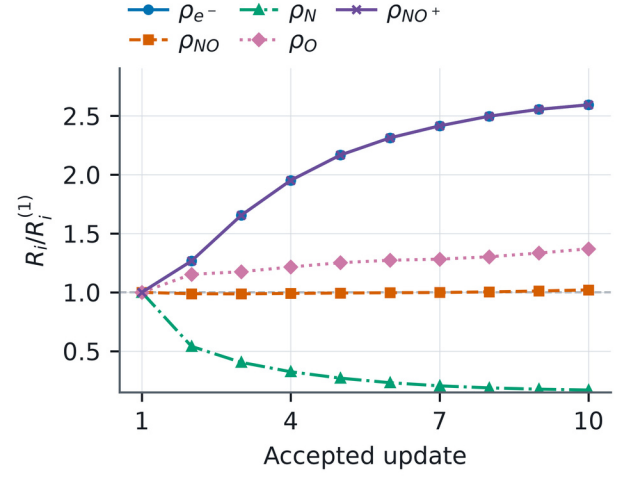


Fig. 25 Localized intake evolution within $0.10 \leq x \leq 0.23$ and $0.02 \leq z \leq 0.15$ m on $y = 0$. Fine iteration 10 has $p_{\min} = 1517.98$ Pa, $T_{\min} = 209.360$ K, and $M_{\max} = 6.24720$, compared with 536.955 Pa, 61.6348 K, and 11.9002 in the donor window. Extrema use each mesh's native section and need not coincide spatially. The inherited local extreme relaxes substantially, while colder global extrema persist elsewhere.

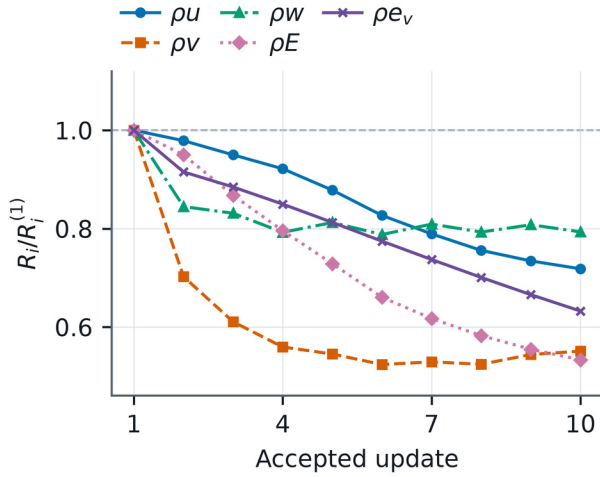
Molecular species



Trace and charged species



Momentum and energies



SST turbulence

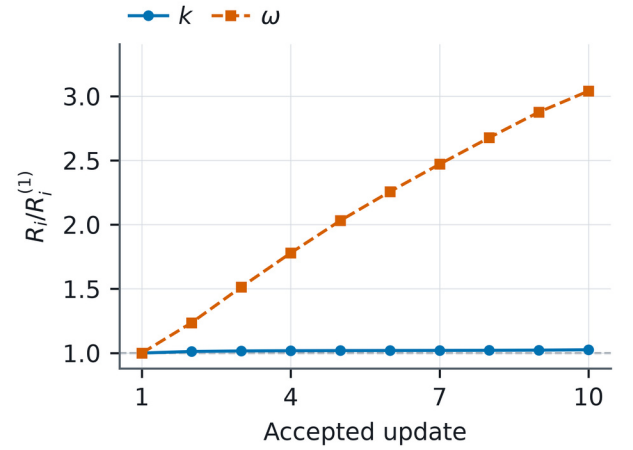


Fig. 26 Fourteen equation-wise native assembly RMS histories through accepted fine iterations 1–10, normalized by each equation's first reported norm. This displays relative progression independently of dimensional scale. FLOW and SST values come from their respective native assembly phases; the short, mixed relaxation history is not a simultaneous final-state nonlinear-defect reassembly or evidence of established convergence. Absolute norms appear in Fig. 39.

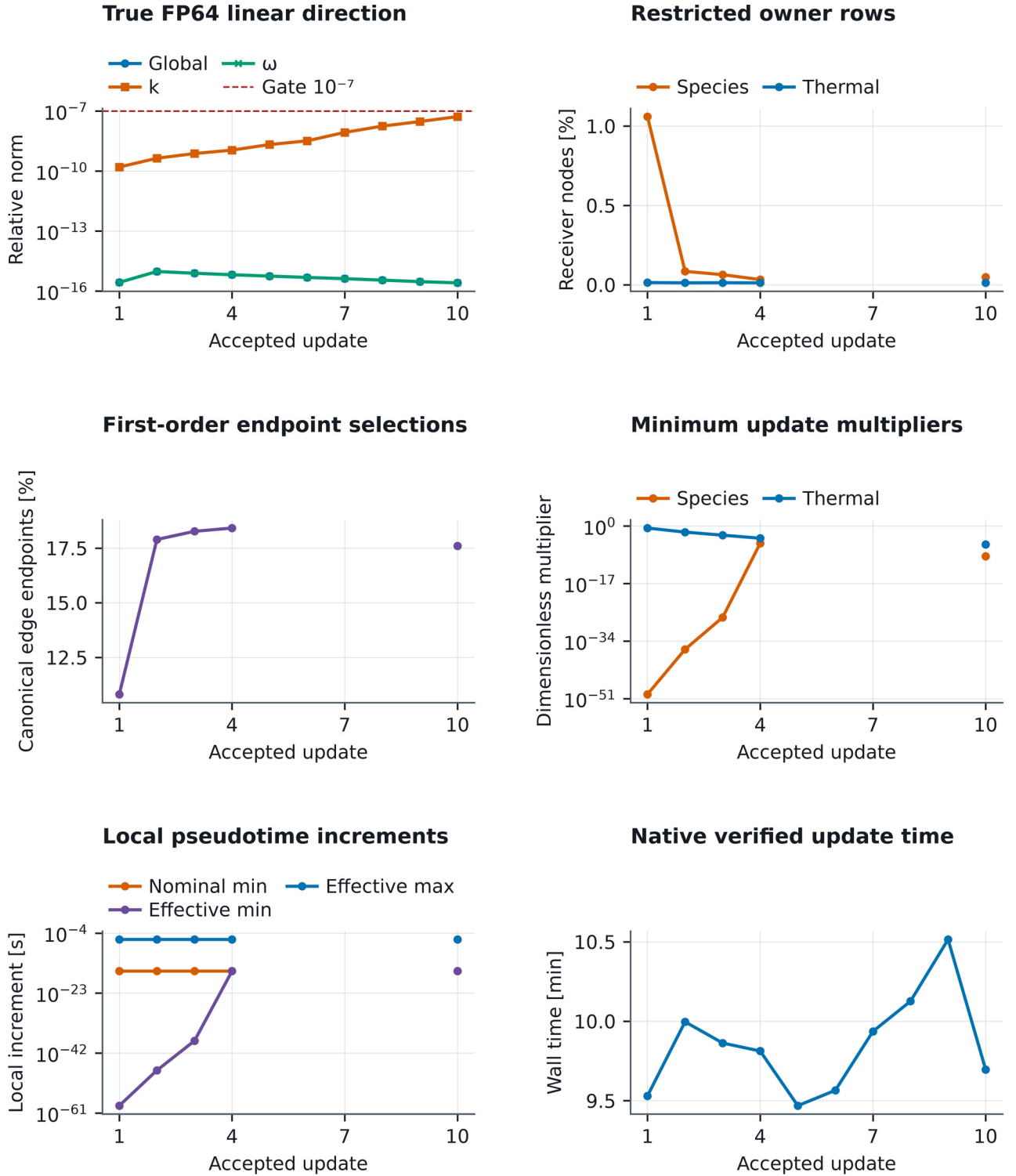


Fig. 27 Native linear-direction acceptance, local update restrictions, reconstruction selections, local pseudo-time spans, and measured update cost through fine iteration 10 at fixed nominal CFL 0.5. Reconstruction counts use $2E = 253,509,808$ canonical edge-endpoint visits; restricted owner-row fractions use 42,396,569 nodes. Counter records exist for iterations 1–4 and 10, and missing counters are not interpolated. Pseudo-time increments do not measure physical flow-development time.

I. Supplementary Fine-Grid Field and Numerical Diagnostics

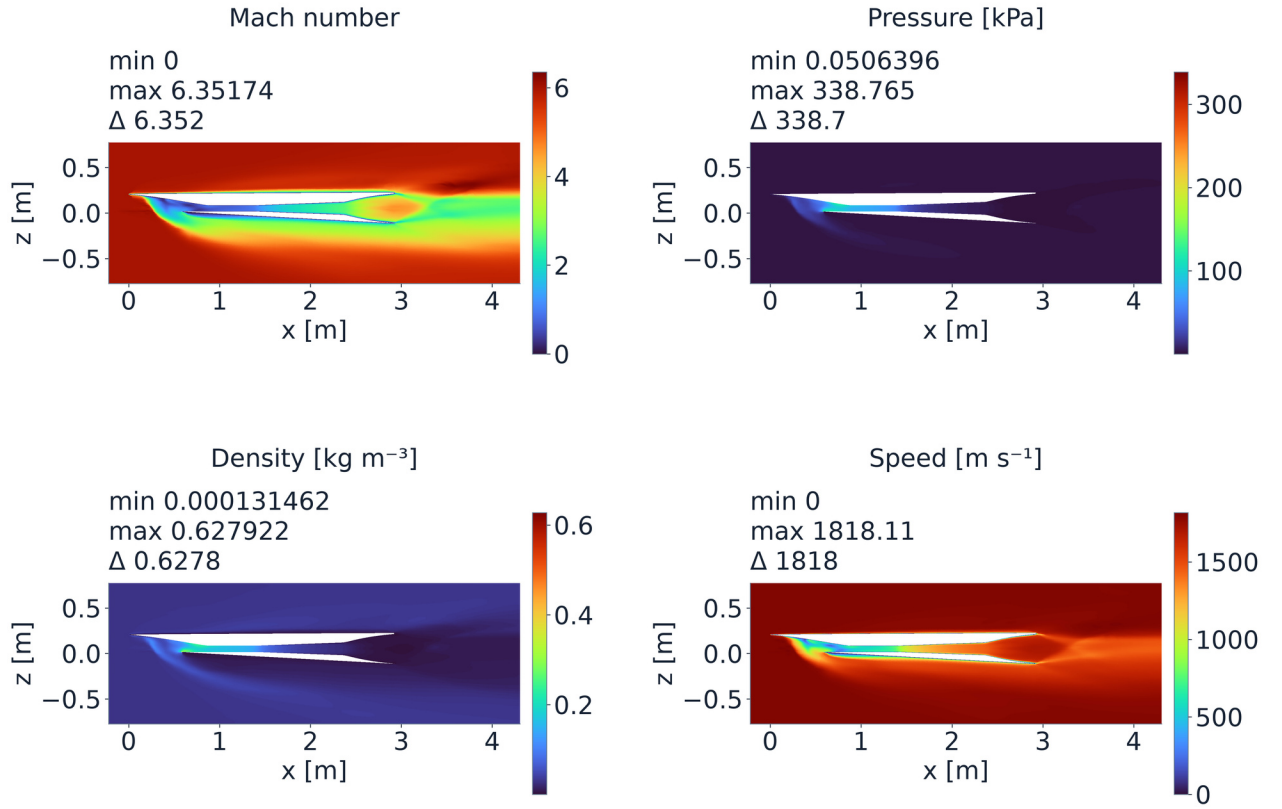
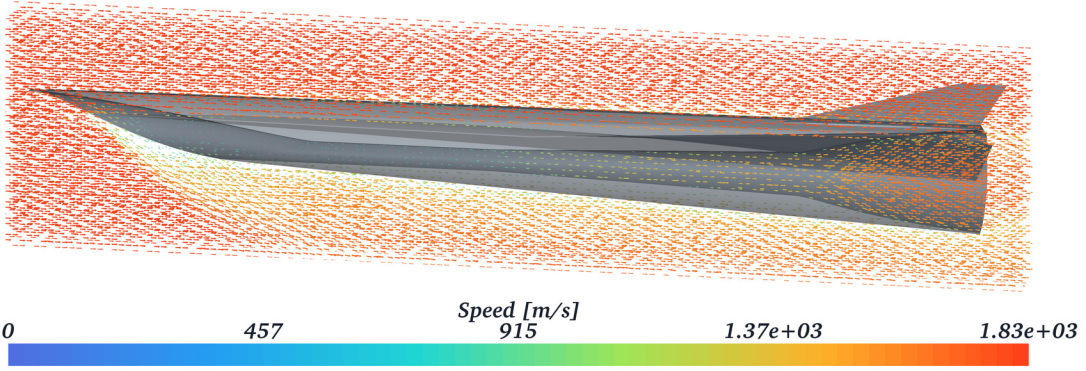


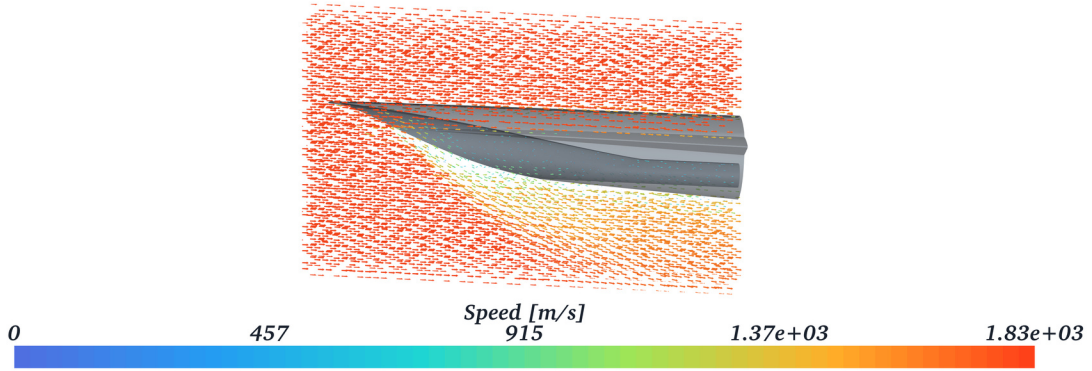
Fig. 28 Mach number, pressure, density, and speed on the complete receiver center-plane fluid section at fine iteration 10. Density is the sum of species densities and velocity is $Q_1(m)/Q_1(\sum_s \rho_s)$ in the containing native hexahedron. Native two-temperature recovery supplies pressure and acoustic quantities; each panel retains its own declared physical scale.

Full vehicle

x = -0.060 to 3.072 m

**Intake turning**

x = -0.060 to 1.133 m

**Aft body / retained downstream cells**

x = 2.147 to 3.728 m

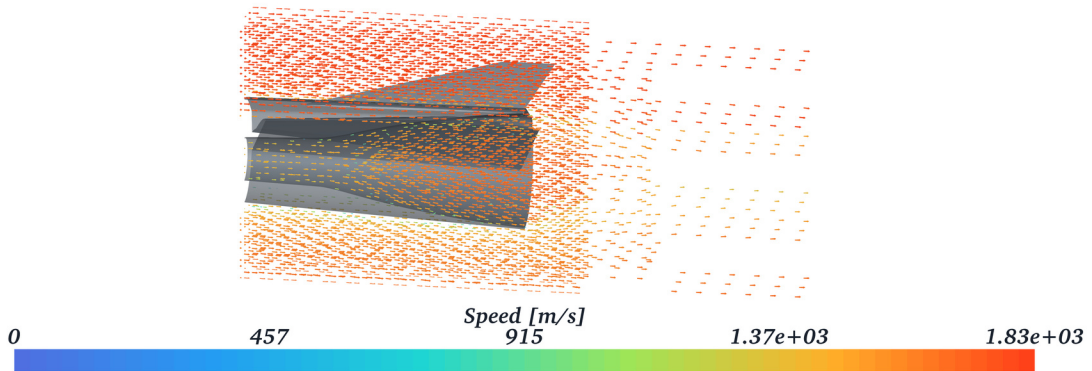
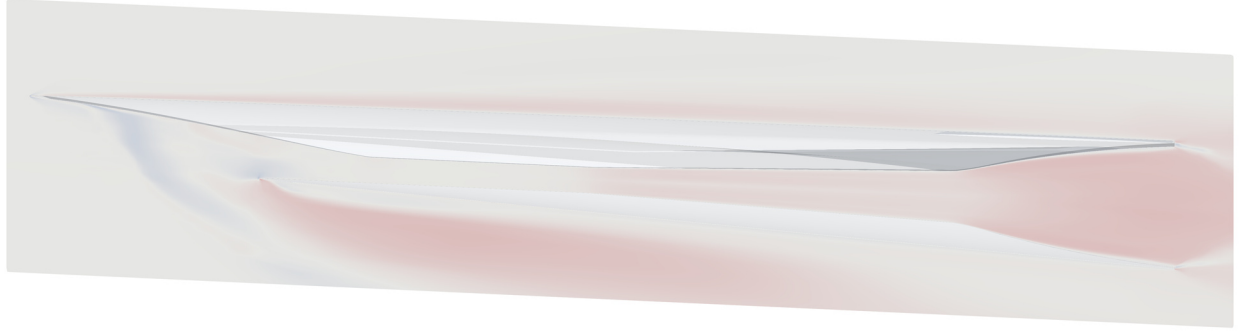


Fig. 29 Conservative-velocity vectors in the retained vertical receiver bands at $y = 0$ and $y = \pm 0.0825$ m, shown at vehicle, intake, and aft-body scales for fine iteration 10. Every arrow uses native containing-cell interpolation. A common length scale $\ell = (10^{-5} \text{ s})|u|$ preserves relative speed; color independently reports the same physical magnitude. Arrow cross-sectional widths are enlarged for printed visibility; their directions and speed-scaled lengths are unchanged.

$T_{ve} - T_{tr}$ [K]: full vehicle
 $x = -0.060$ to 3.072 m

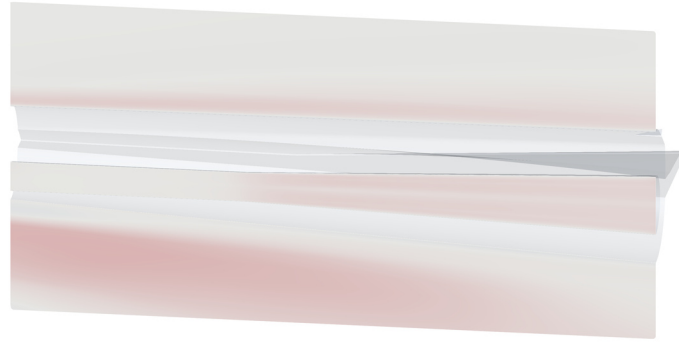
$\frac{4}{X}$



$T_{ve} - T_{tr}$ [K]: full vehicle
 $-2.27e+03$ $-1.14e+03$ 0 $1.14e+03$ $2.27e+03$

$T_{ve} - T_{tr}$ [K]: isolator / aft
 $x = 0.895$ to 2.356 m

$\frac{4}{X}$



$T_{ve} - T_{tr}$ [K]: isolator / aft
 $-1.82e+03$ -910 0 910 $1.82e+03$

Fig. 30 Colocated $T_{ve} - T$ on the retained vertical receiver section at fine iteration 10, at vehicle and isolator/aft-body scales. Symmetric signed color scales distinguish internal-mode temperature above or below the translational-rotational temperature. Local source terms, rather than a temperature difference alone, determine relaxation rates.

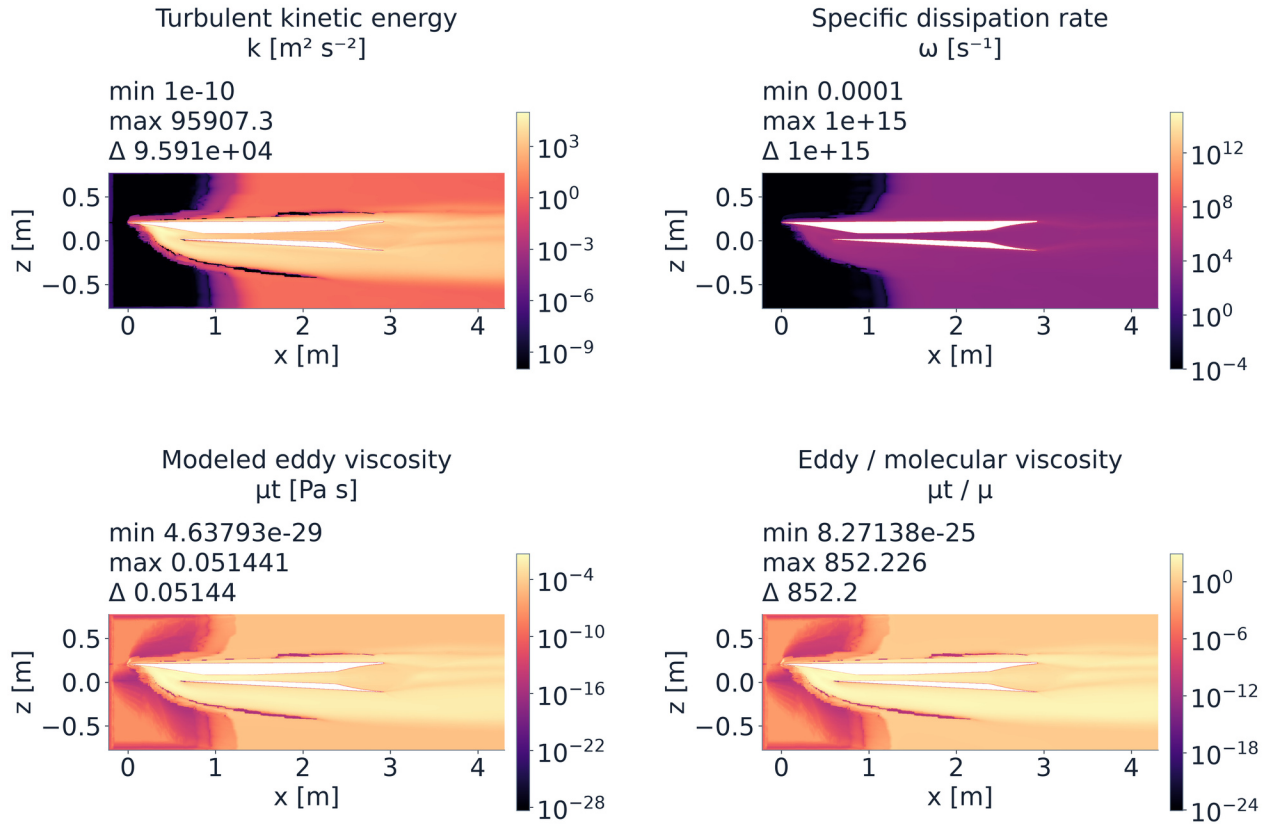


Fig. 31 Native SST variables and turbulent transport at fine iteration 10 on the receiver $y = 0$ section: k , ω , stored μ_t , and the colocated viscosity ratio μ_t/μ . Individual logarithmic scales expose spatial dynamic ranges and bound activity. Linear-direction accuracy and nonlinear assembly progression are assessed separately in the residual figures.

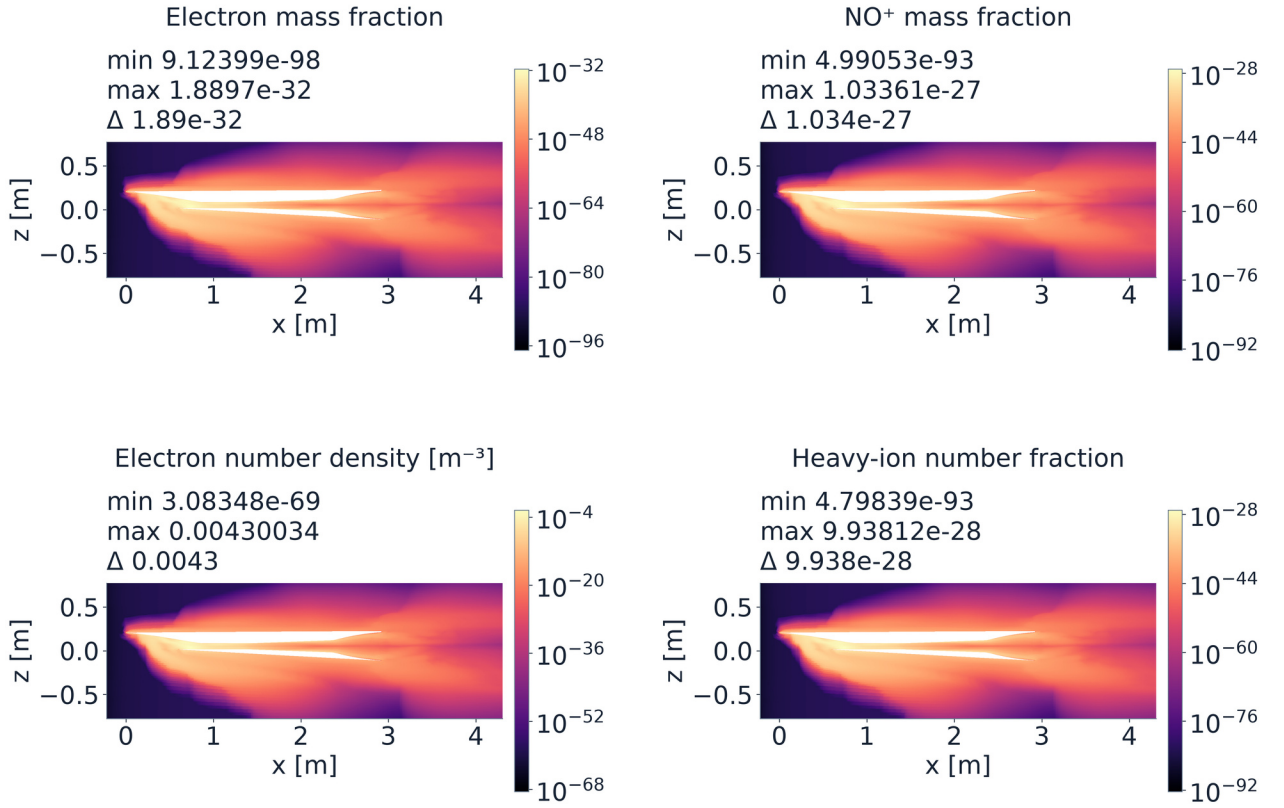


Fig. 32 Trace charged-species fields on the fine receiver center-plane section at iteration 10, using the native species ordering, molar-mass convention, and charge-consistent number-density conversion. Logarithmic color scales expose weak abundance over a wide dynamic range. The full-domain electron-density maximum is only 0.400054 m^{-3} ; display contrast does not imply a substantial plasma.

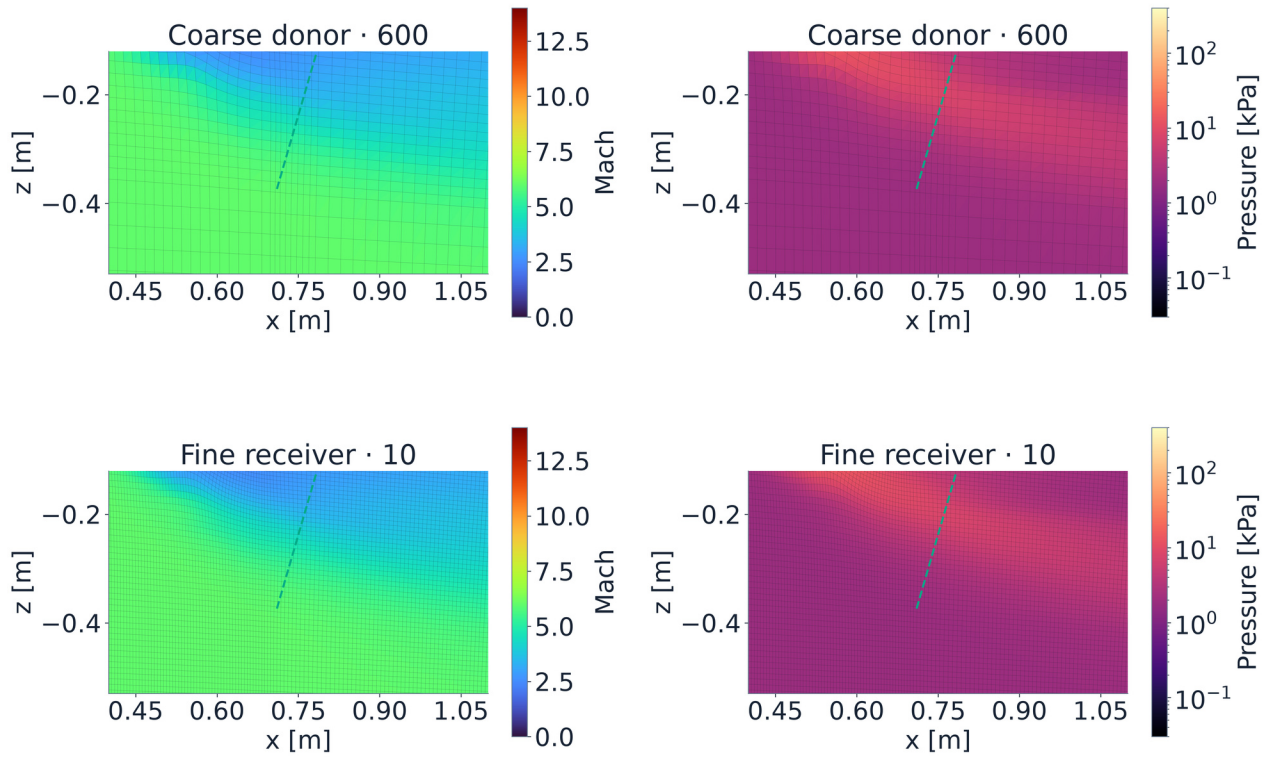


Fig. 33 Matched lower-compression sections for the coarse donor and fine receiver, retaining each mesh's native cell intersections and coordinates. The visible compound compression is evaluated quantitatively along the common three-dimensional lower transect and its nearby sensitivity probes.

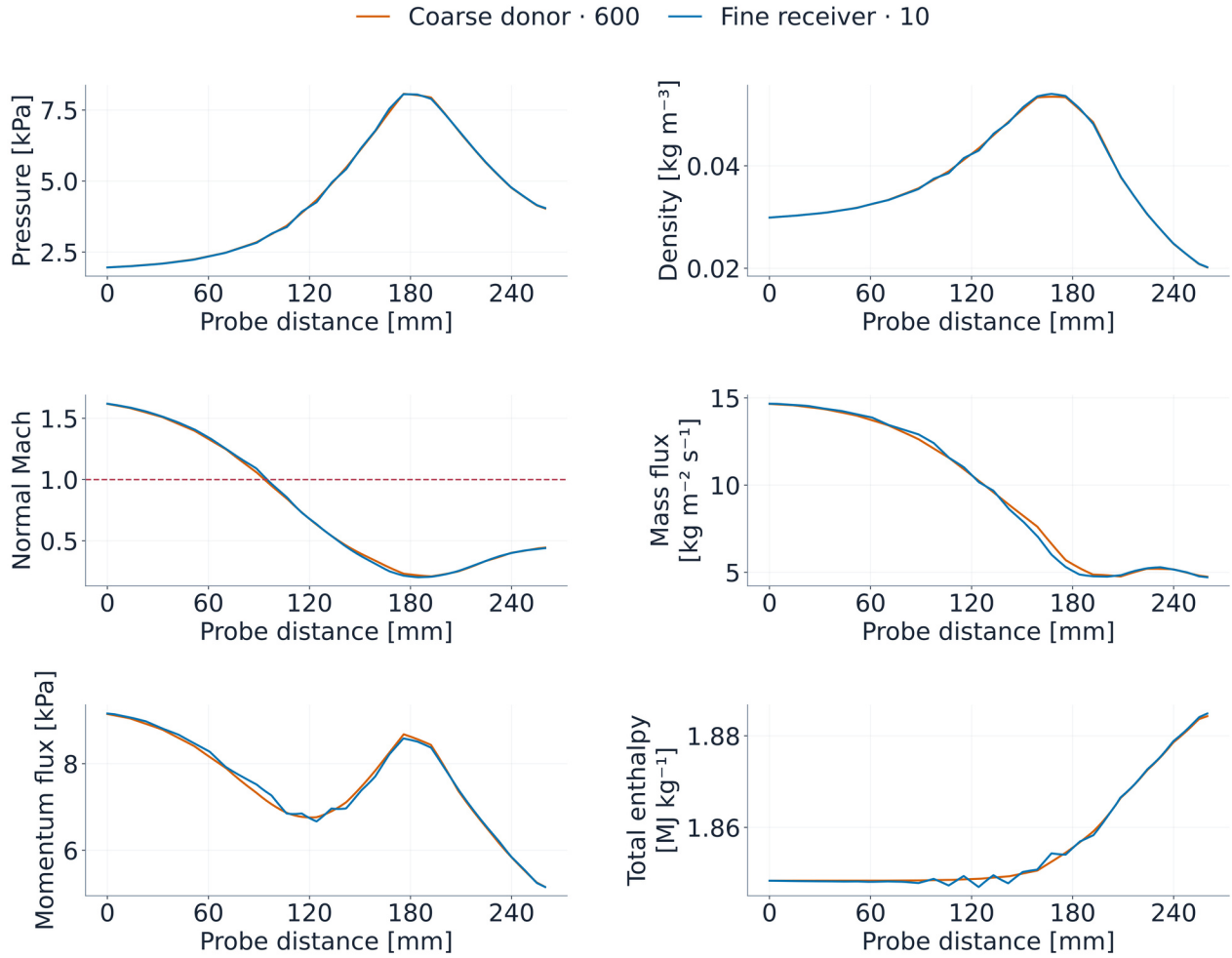


Fig. 34 Common-coordinate lower-compression profiles at donor iteration 600 and fine iteration 10. Receiver and donor compound pressure ratios are 4.11028 and 4.11586; corresponding 10–90% rise widths are 91.1853 and 92.9453 mm. Fifteen and nine distinct native cells are hit by samples within those intervals. These widths describe compound numerical compression rather than physical shock thickness.

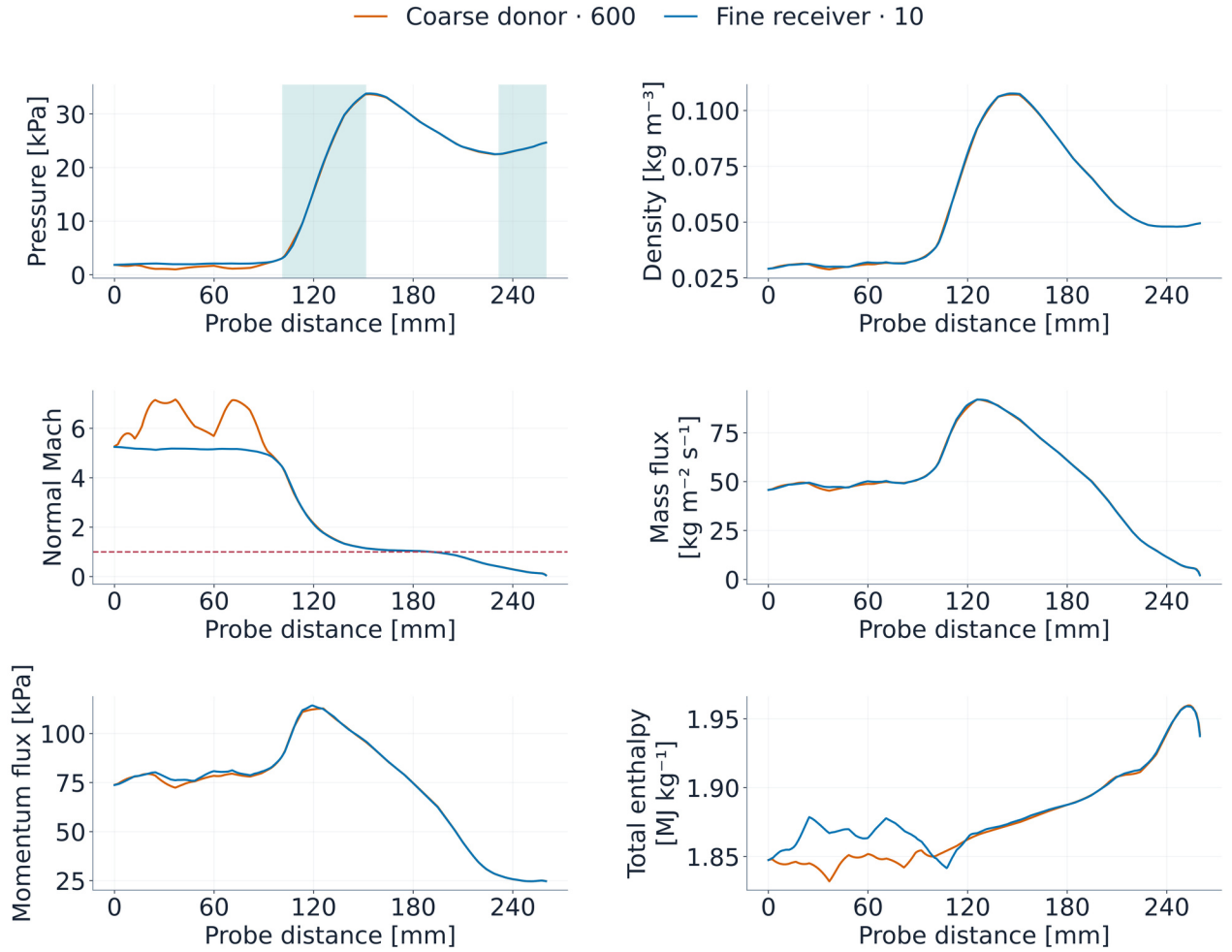


Fig. 35 Matched center-plane intake profiles on the explicitly projected $y = 0$ transect. This line differs geometrically from the local three-dimensional pressure-normal transect. Conservative momentum/density sampling and native-cell containment are retained for both fields; donor acoustic interpolation retains its stated output-precision limitation.

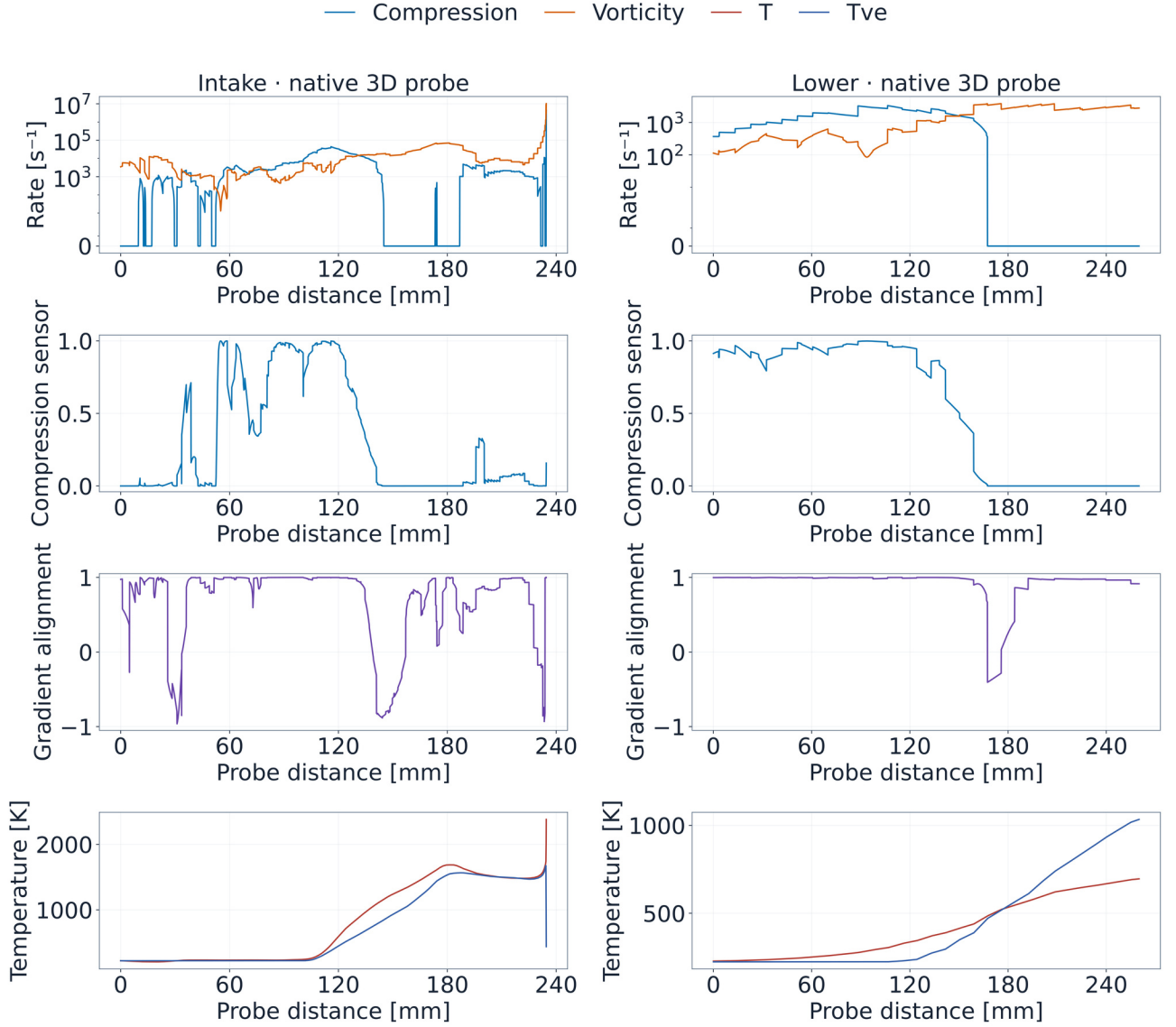


Fig. 36 Fine-iteration-10 compression and thermal diagnostics along the intake and lower three-dimensional transects. Rows report compression $\max(-\nabla \cdot u, 0)$ and rotation $|\nabla \times u|$; their compression-squared fraction; pressure-density gradient alignment; and colocated T and T_{ve} . Derivatives and validity use the actual containing native cell. These receiver-only profiles complement the matched donor/receiver state comparisons.

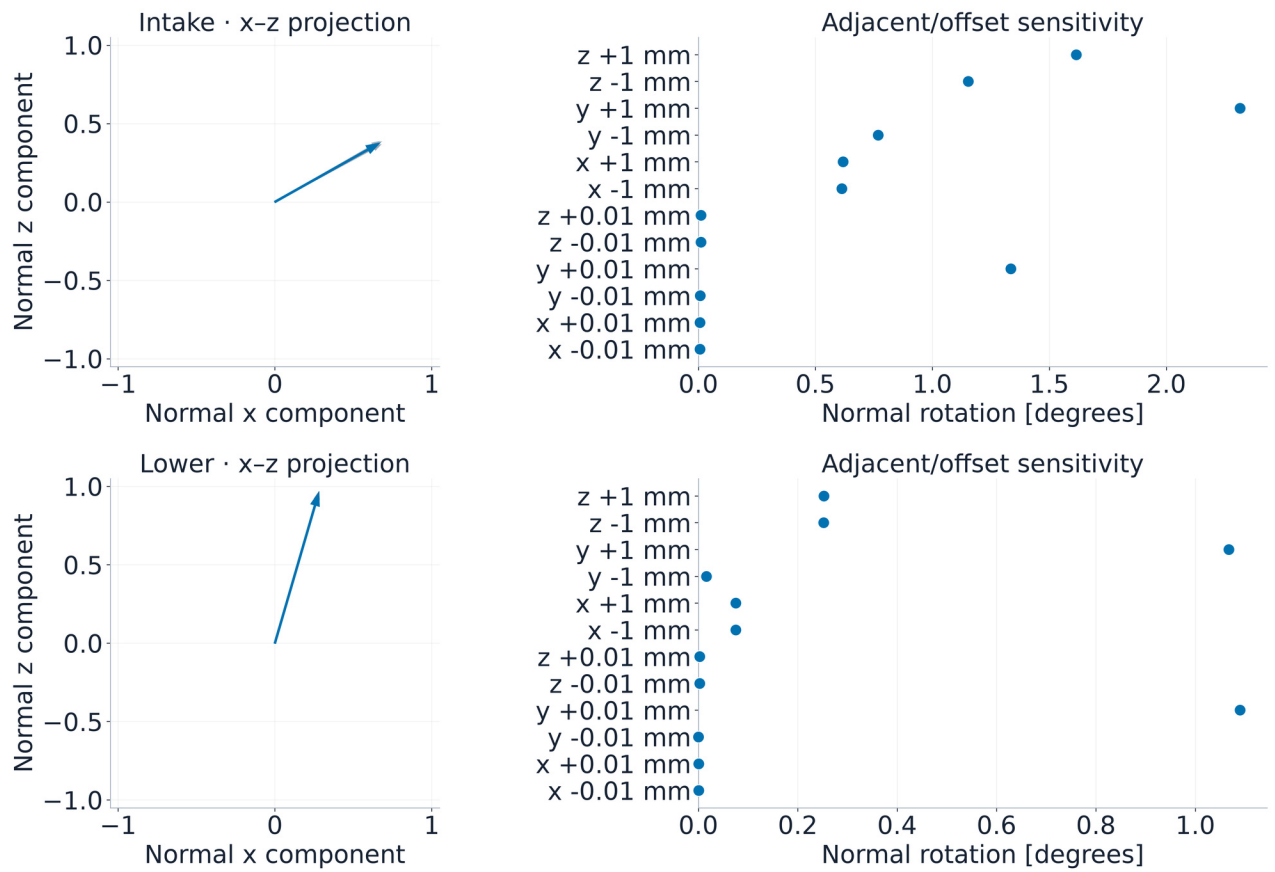


Fig. 37 Sensitivity of the native Q1 pressure-gradient direction to twelve nearby offsets at each anchor. The maximum fine-iteration-10 direction change is 2.3152° at the intake and 1.0900° at the lower anchor. A containing-cell derivative defines each normal; this finite local test does not replace a globally smooth shock-surface construction.

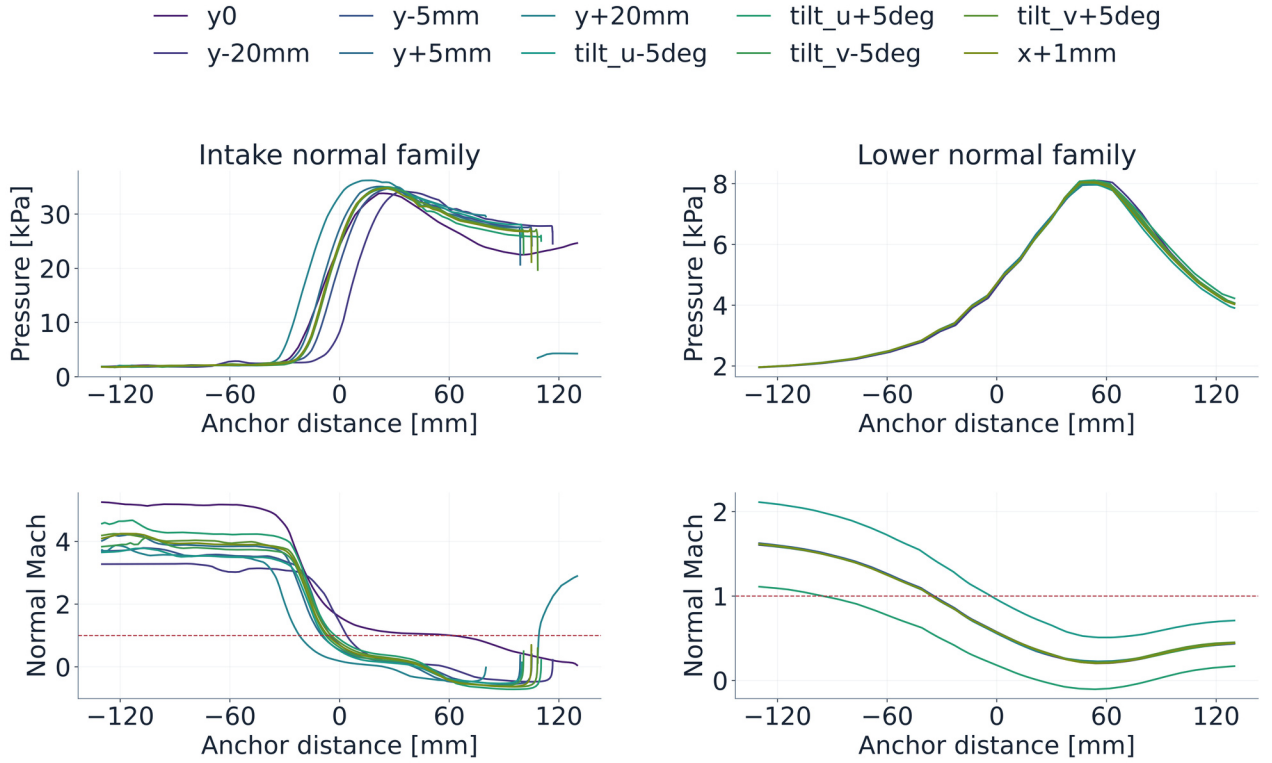


Fig. 38 Receiver-only pressure and projected normal-Mach sensitivity at fine iteration 10 for ten nearby intake lines and ten lower-compression lines. Each family contains the projected center-plane line, spanwise offsets of ± 5 and ± 20 mm, rotations of $\pm 5^\circ$ about each declared transverse direction, and a 1-mm streamwise shift. The two central three-dimensional normals are reported separately. All 22 matched donor/receiver comparisons are retained in the numerical report; this figure isolates receiver family sensitivity.

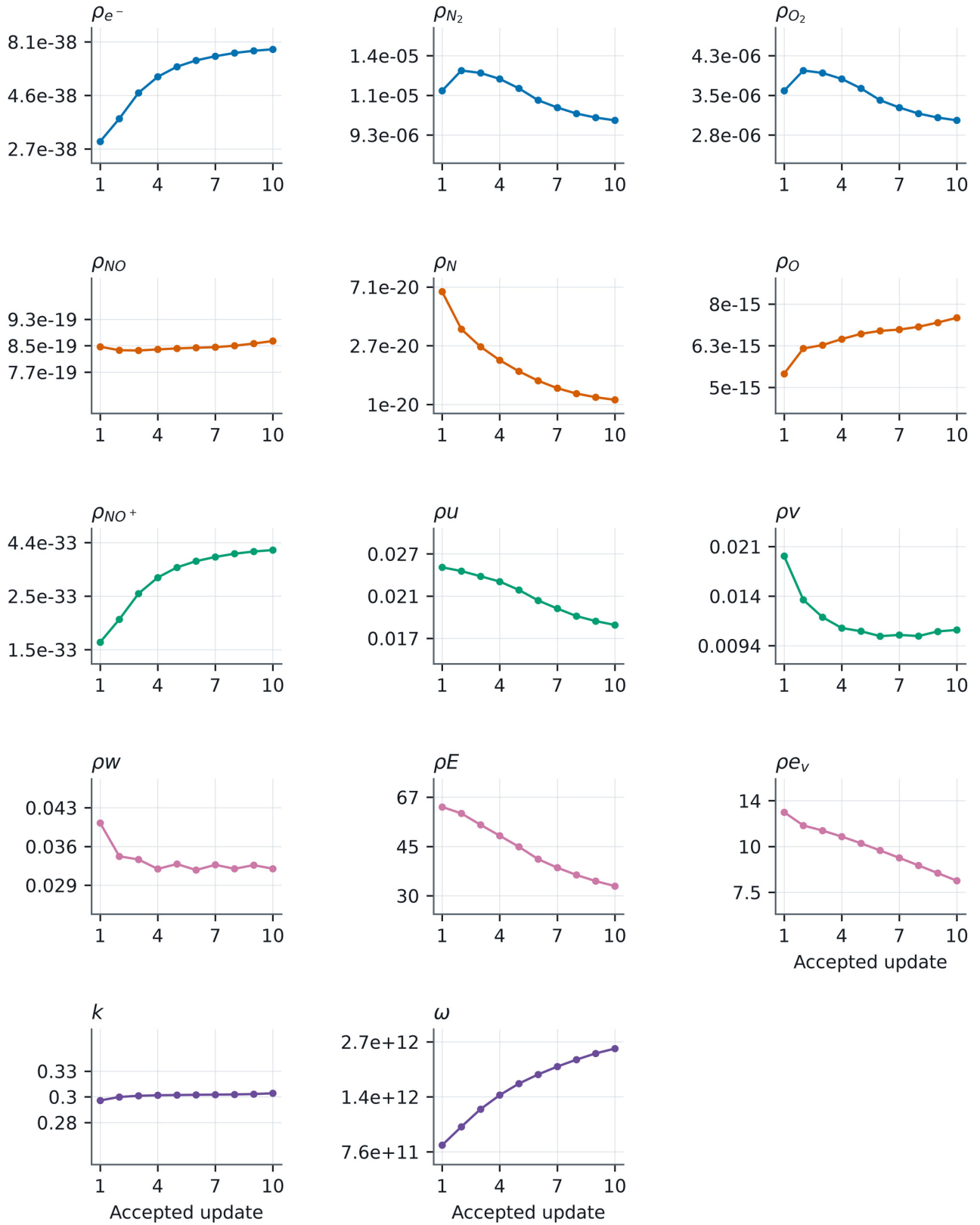


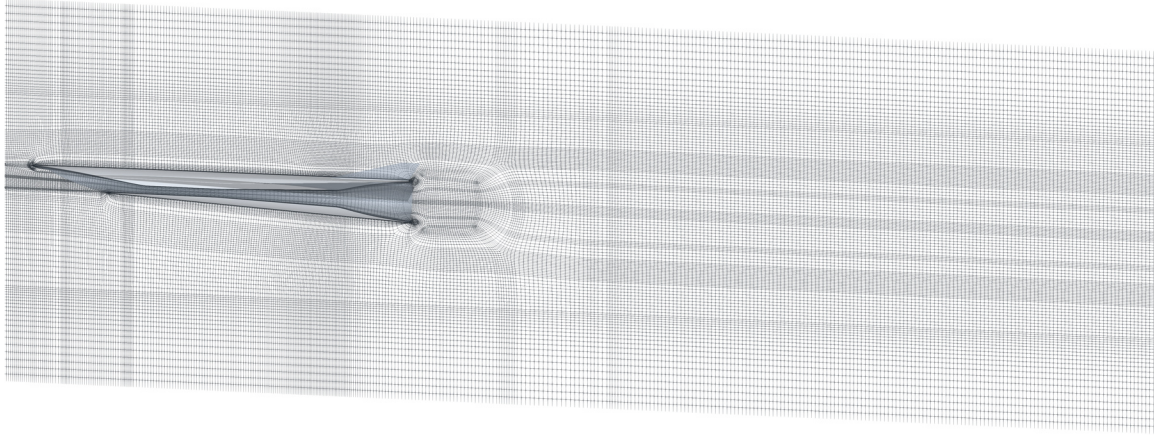
Fig. 39 Absolute native phase-assembly RMS norms for all fourteen equations through fine iterations 1–10. Each equation retains its dimensional magnitude, including the very small charged-species scales. The values describe the separate native FLOW and SST assembly phases; they are not a newly reassembled common-state nonlinear defect.

II. Retained Donor Field and Compression Diagnostics

The following plates retain donor iteration 600 and its original-grid baseline comparisons. Their fields precede the fine-grid transfer and are distinct from the ten-update receiver results.

Coarse donor mesh: 3.999 million hexahedra

Native x range: -0.22 to 8.70 m



Intake and compression-region cell allocation

Native x range: -0.10 to 1.05 m

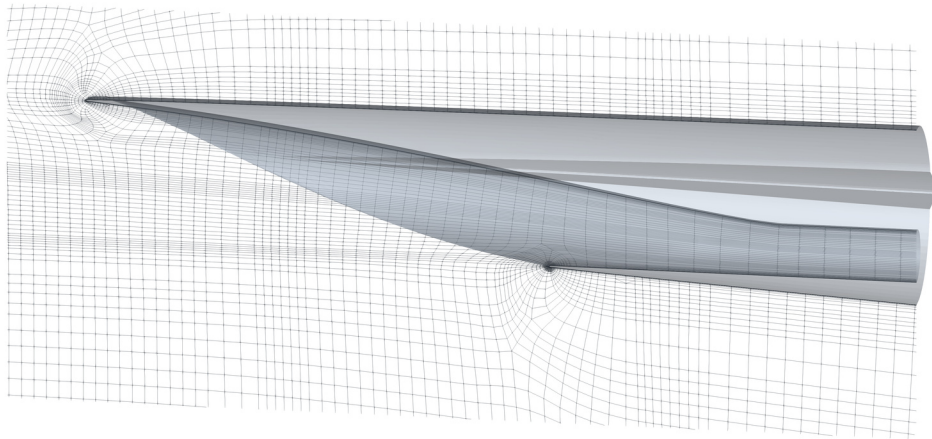
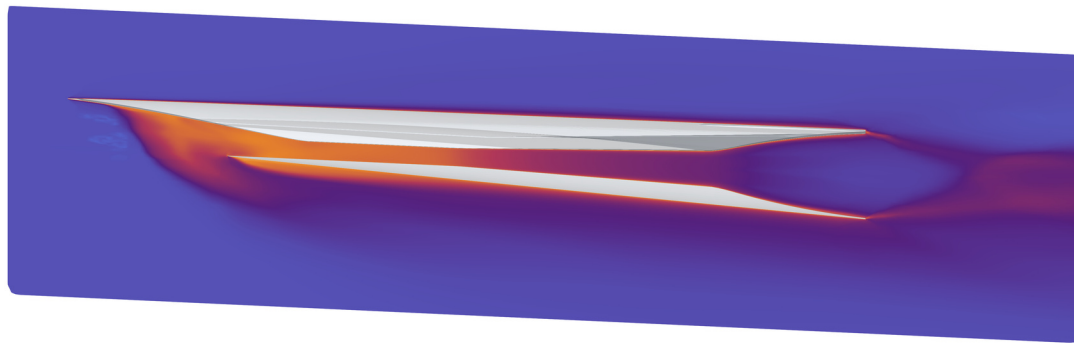


Fig. 40 Coarse donor mesh: native $y = 0$ fluid-cell intersection edges and physical walls, shown across the extended domain and intake with unchanged FP64 coordinates.

Translation / rotation temperature [K]

Native x range: -0.22 to 3.70 m

$\frac{4}{X}$

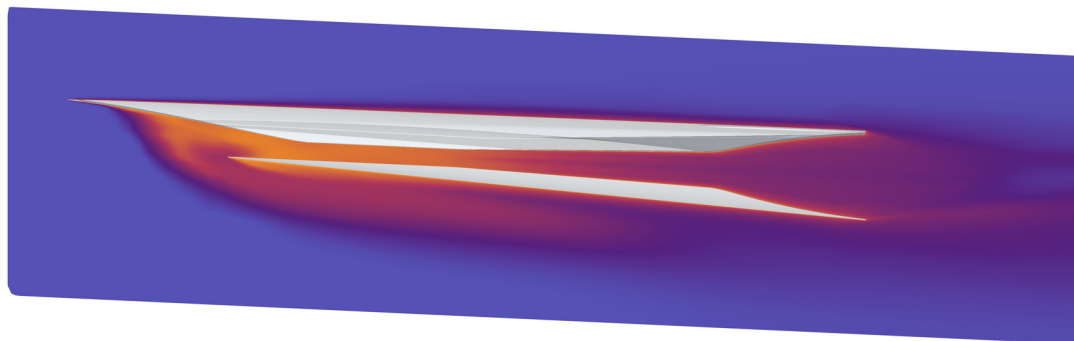


50 Translation / rotation temperature [K] 818 1.59e+03 2.36e+03

Vibration / electronic temperature [K]

Native x range: -0.22 to 3.70 m

$\frac{4}{X}$

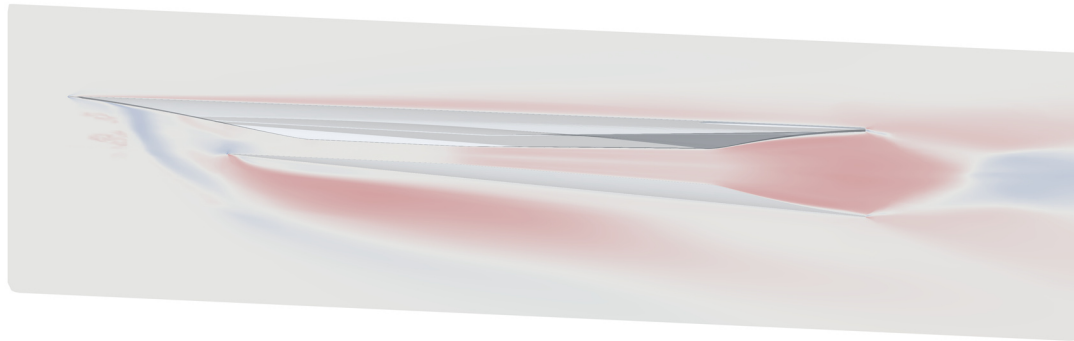


50 Vibration / electronic temperature [K] 818 1.59e+03 2.36e+03

Fig. 41 Donor-600 translational/rotational and vibrational/electronic temperatures on the native vertical $y = 0$ section, using common scales and a near-side cutaway of the physical vehicle.

$T_{ve} - T_{tr}$ [K]: vehicle and near wake
Native x range: -0.22 to 3.70 m

$\frac{4}{X}$



$T_{ve} - T_{tr}$ [K]: intake detail
Native x range: -0.10 to 1.05 m

$\frac{4}{X}$

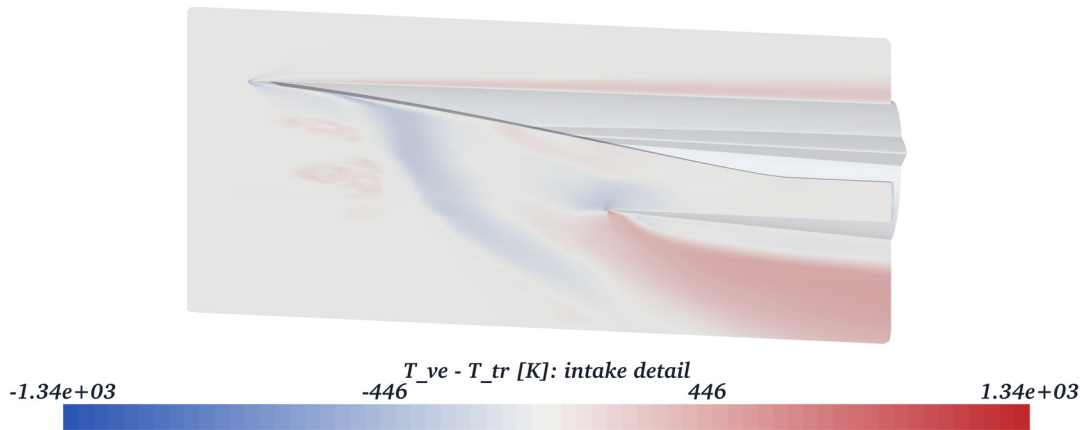
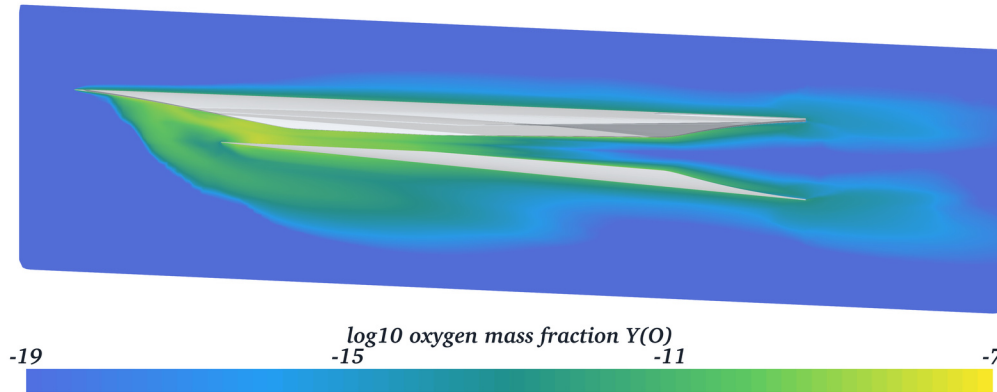


Fig. 42 Donor-600 signed temperature difference $T_{ve} - T_{tr}$ on the native $y = 0$ section: whole-vehicle and intake views share one symmetric color range centered on zero.

log10 oxygen mass fraction $Y(O)$

Native x range: -0.22 to 3.70 m

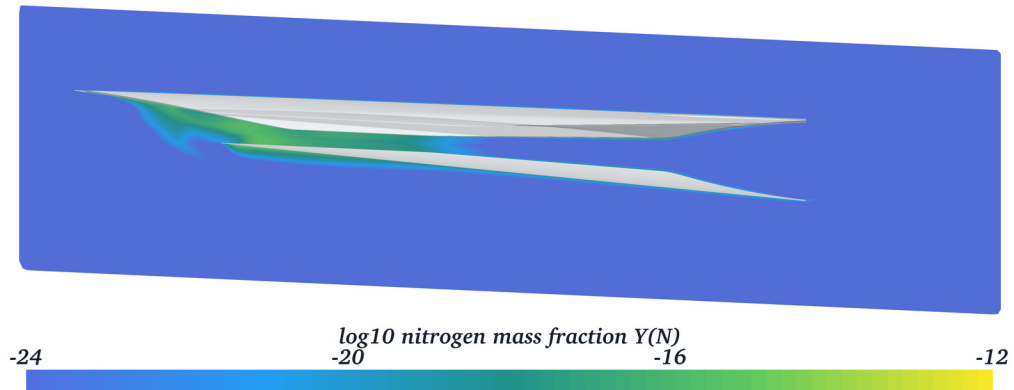
4X



log10 nitrogen mass fraction $Y(N)$

Native x range: -0.22 to 3.70 m

4X



log10 nitric-oxide mass fraction $Y(NO)$

Native x range: -0.22 to 3.70 m

4X

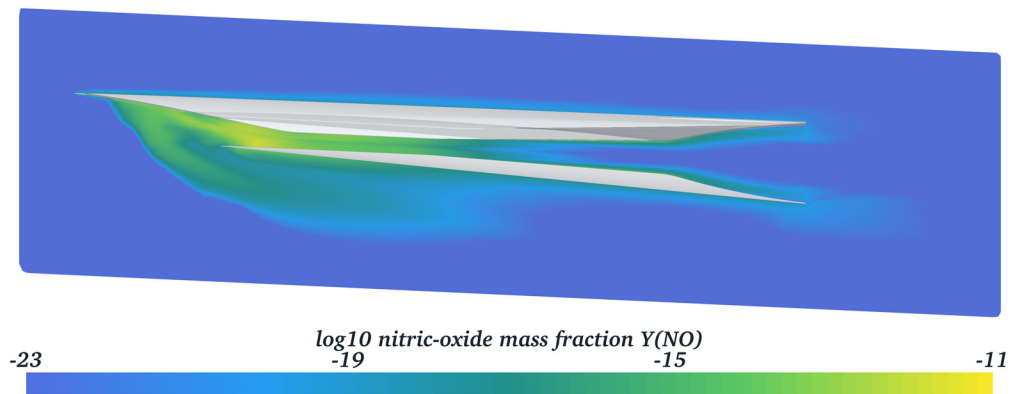
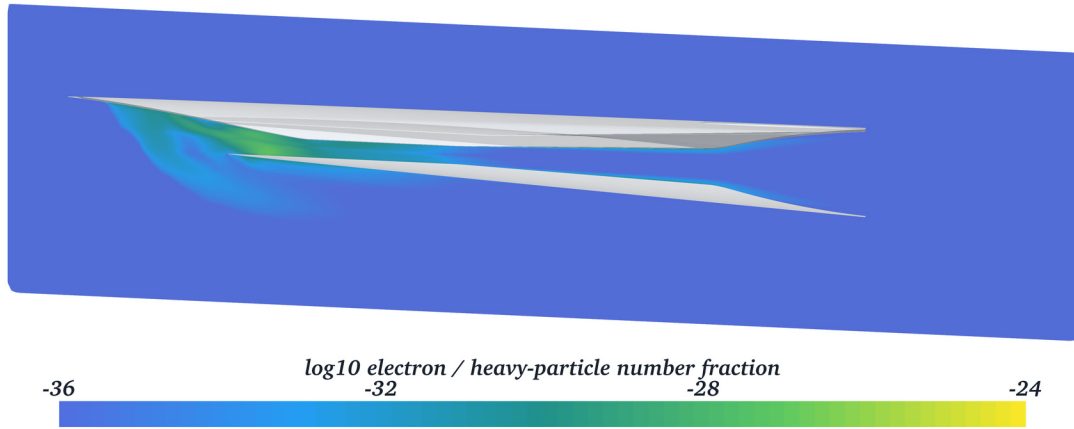


Fig. 43 Donor-600 O, N, and NO mass fractions on the native $y = 0$ section, recovered from interpolated conserved densities; logarithmic display saturation preserves stored concentrations.

log10 electron / heavy-particle number fraction

Native x range: -0.22 to 3.70 m

$\frac{4}{X}$



log10 electron number density [m^{-3}]

Native x range: -0.22 to 3.70 m

$\frac{4}{X}$

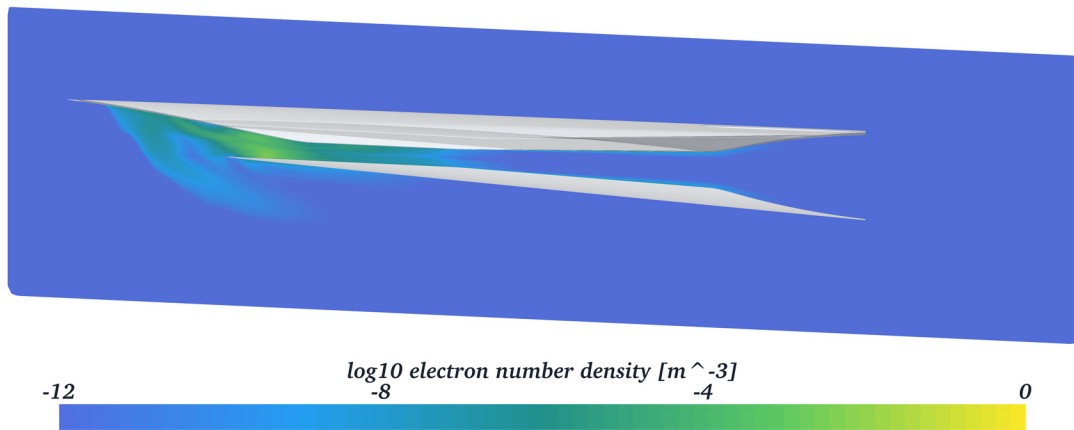
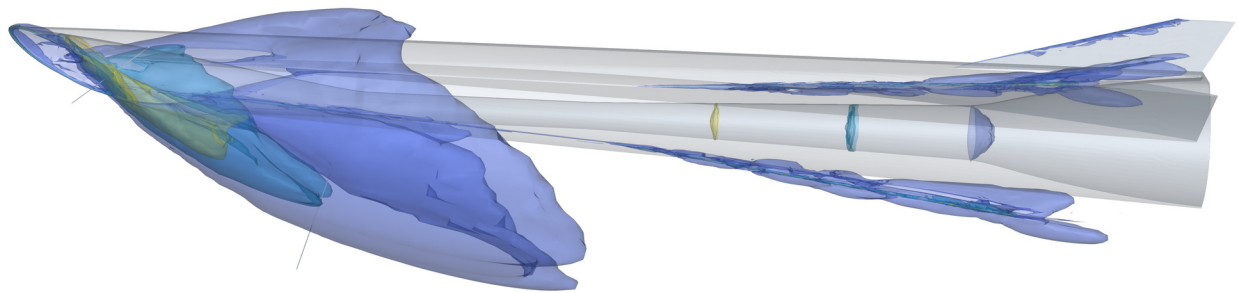


Fig. 44 Donor-600 electron fraction and number density on the native $y = 0$ section, derived from conserved densities; independent logarithmic scales resolve numerically trace charged-species content.

Pressure isosurfaces: 4, 8 and 16 kPa

Native x range: -0.22 to 3.70 m

4X



Intake pressure contours and exact diagnostic transect

Native x range: -0.10 to 1.05 m

4X

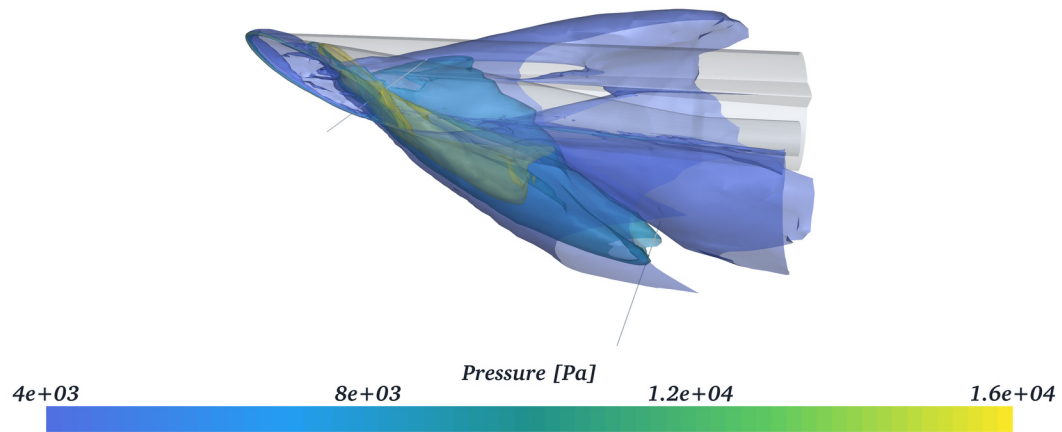
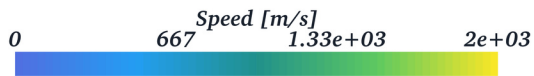
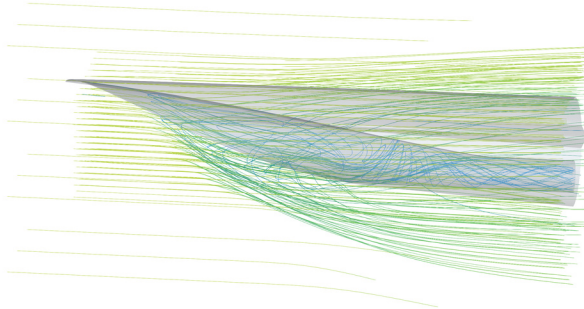


Fig. 45 Donor-600 pressure isosurfaces at 4, 8, and 16 kPa with native diagnostic transects; whole-vehicle and intake views illustrate compression geometry for quantitative profile assessment.

Dense intake / separation-region seeding

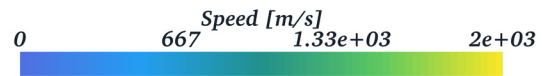
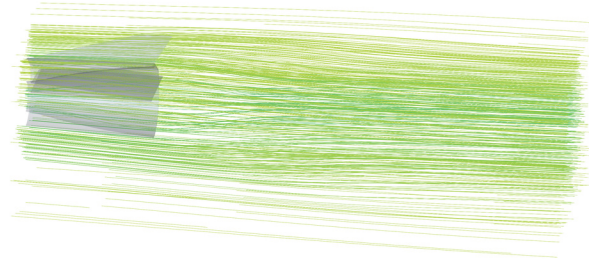
Native x range: -0.10 to 1.05 m



$\frac{4}{X}$

Dense aft-body / near-wake seeding

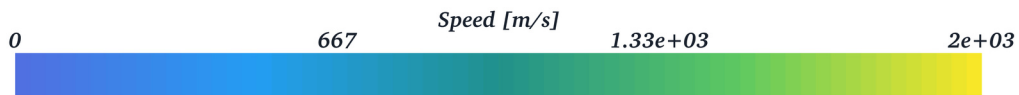
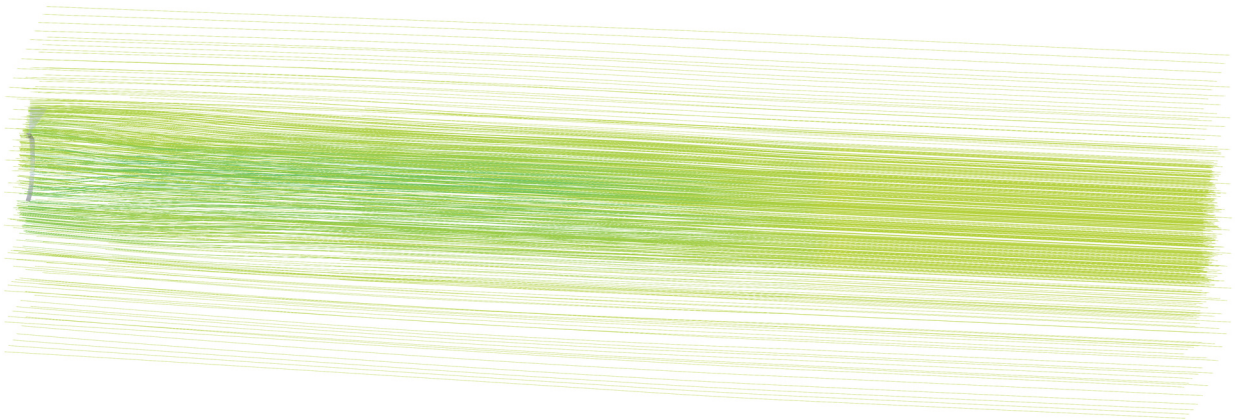
Native x range: 2.35 to 4.80 m



$\frac{4}{X}$

Entire added wake to native outlet: x = 2.90 to 8.70 m

Native x range: 2.90 to 8.70 m



$\frac{Z}{Y} X$

Fig. 46 Donor-600 RK45 velocity integral curves from 552 admitted seeds, using cell-interpolated nodal velocity; intake, near-wake, and extended-domain views retain the saved steady-pseudotime snapshot.

Intake: magnitude-scaled native velocity vectors

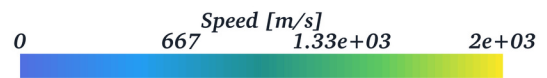
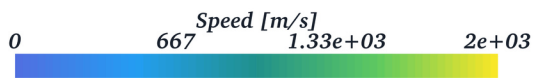
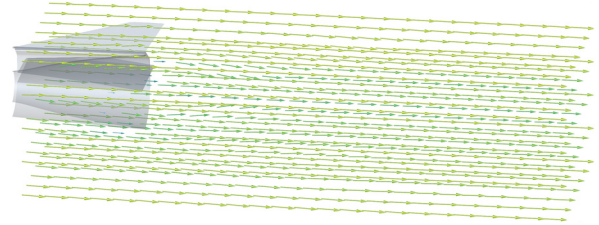
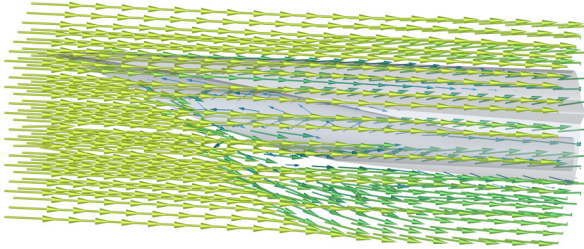
Native x range: -0.10 to 1.05 m

$\frac{4}{X}$

Near wake: same vector-length scale

Native x range: 2.35 to 4.80 m

$\frac{4}{X}$



Entire added wake: $x = 2.90$ to 8.70 m; same physical scale

Native x range: 2.90 to 8.70 m

$\frac{4}{X}$

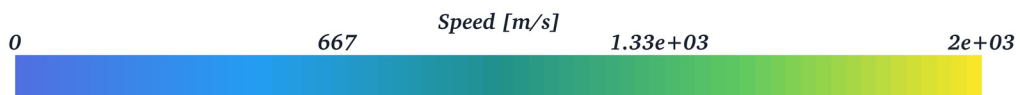
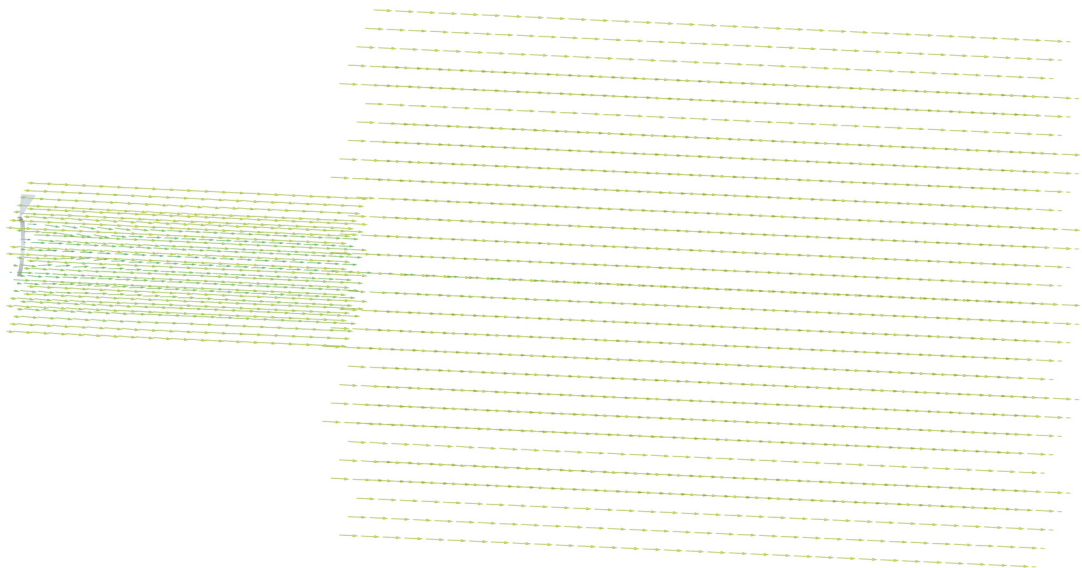


Fig. 47 Donor-600 velocity vectors recovered from interpolated conserved momentum and density. All views share the speed color scale and arrow-length scale of $60 \mu\text{s}$.

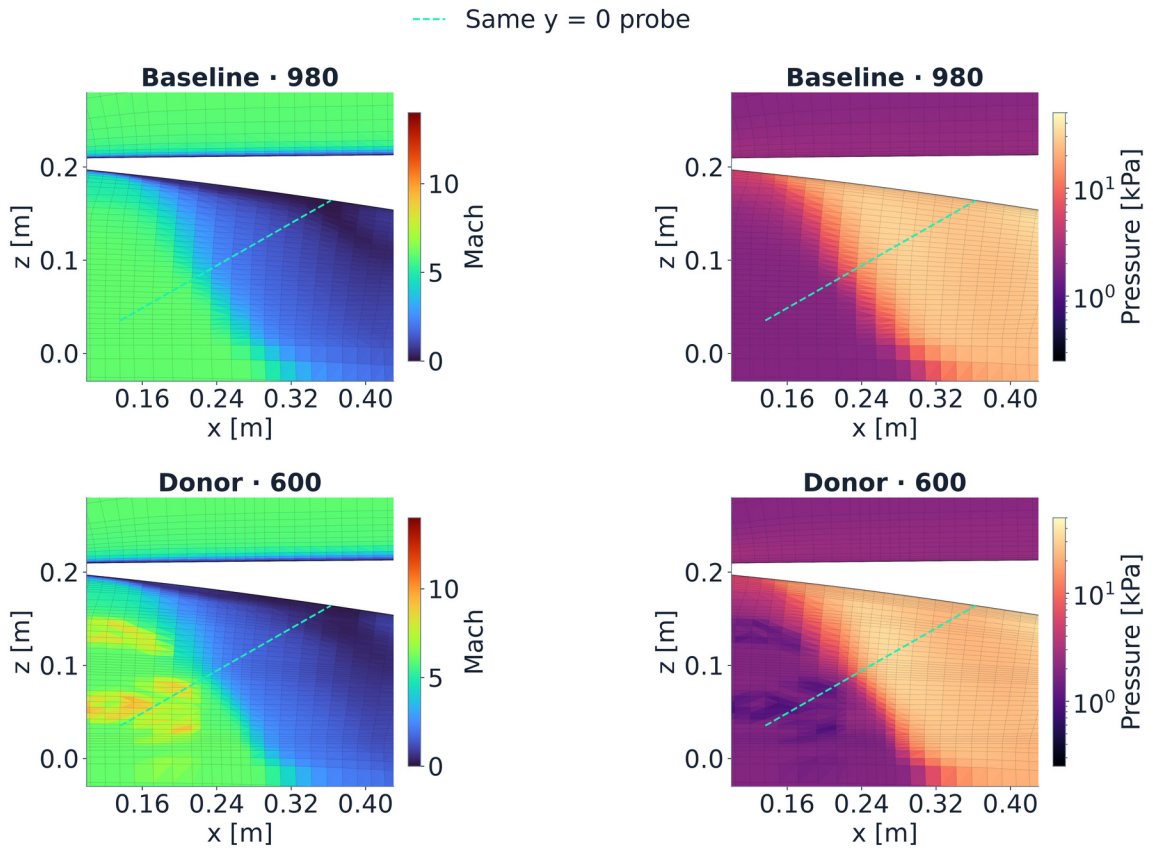


Fig. 48 Historical intake comparison: baseline 980 and donor 600 share spatial windows, Mach and pressure scales, and the $y = 0$ probe; edges retain native cells.

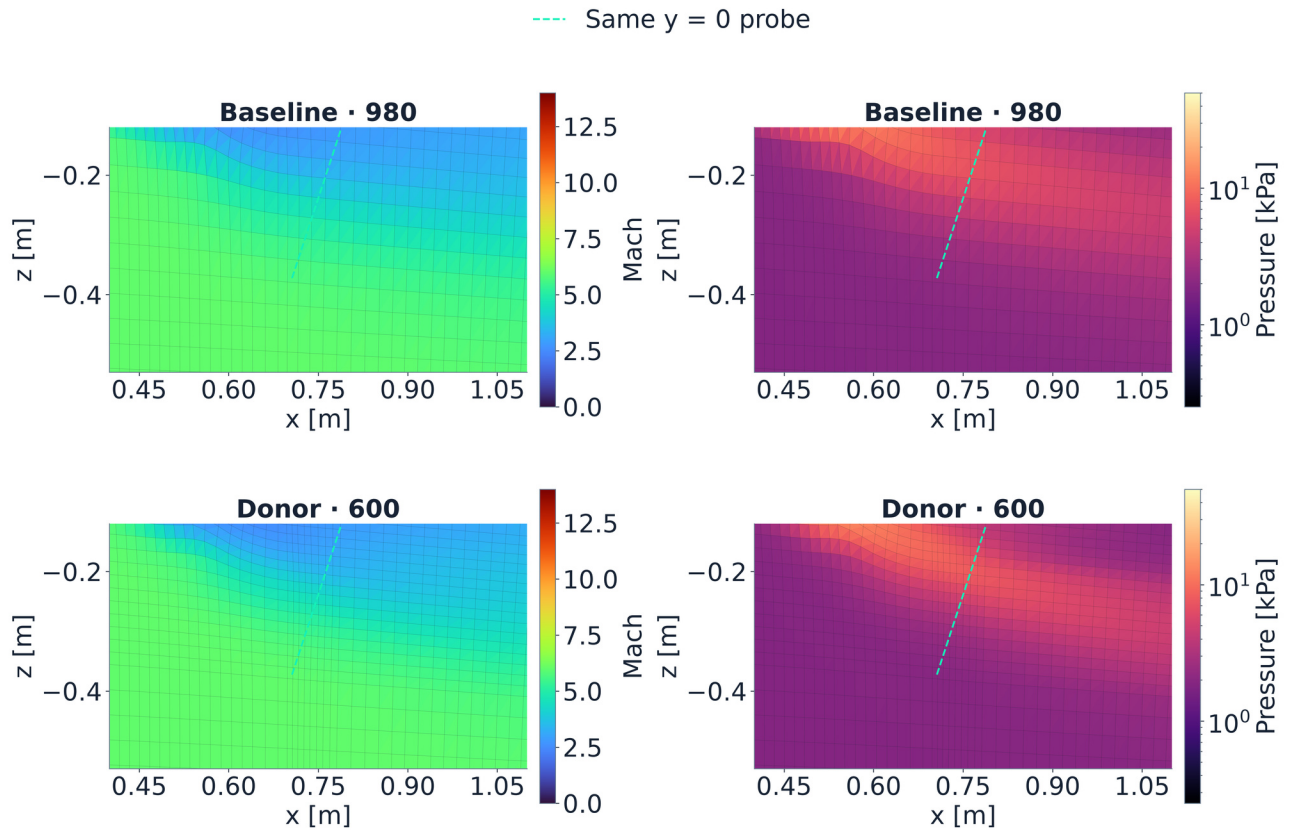


Fig. 49 Historical lower-body comparison: baseline 980 and donor 600 share spatial windows, Mach and pressure scales, and the $y = 0$ probe; edges retain native cells.

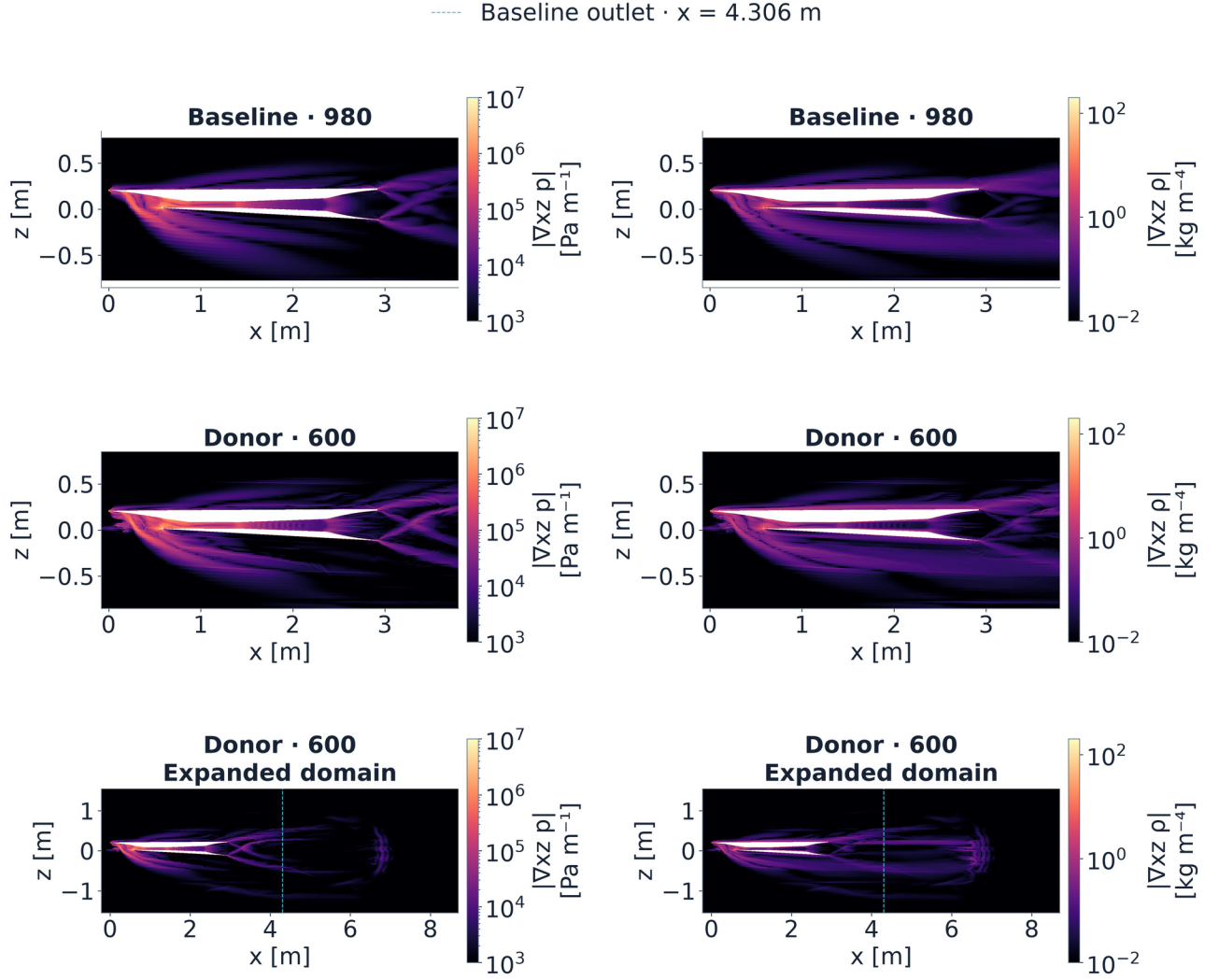


Fig. 50 $y = 0$ sections. Black includes zero and subthreshold gradients; gray is outside fluid. Bottom: complete expanded domain; dashed line: old outlet.

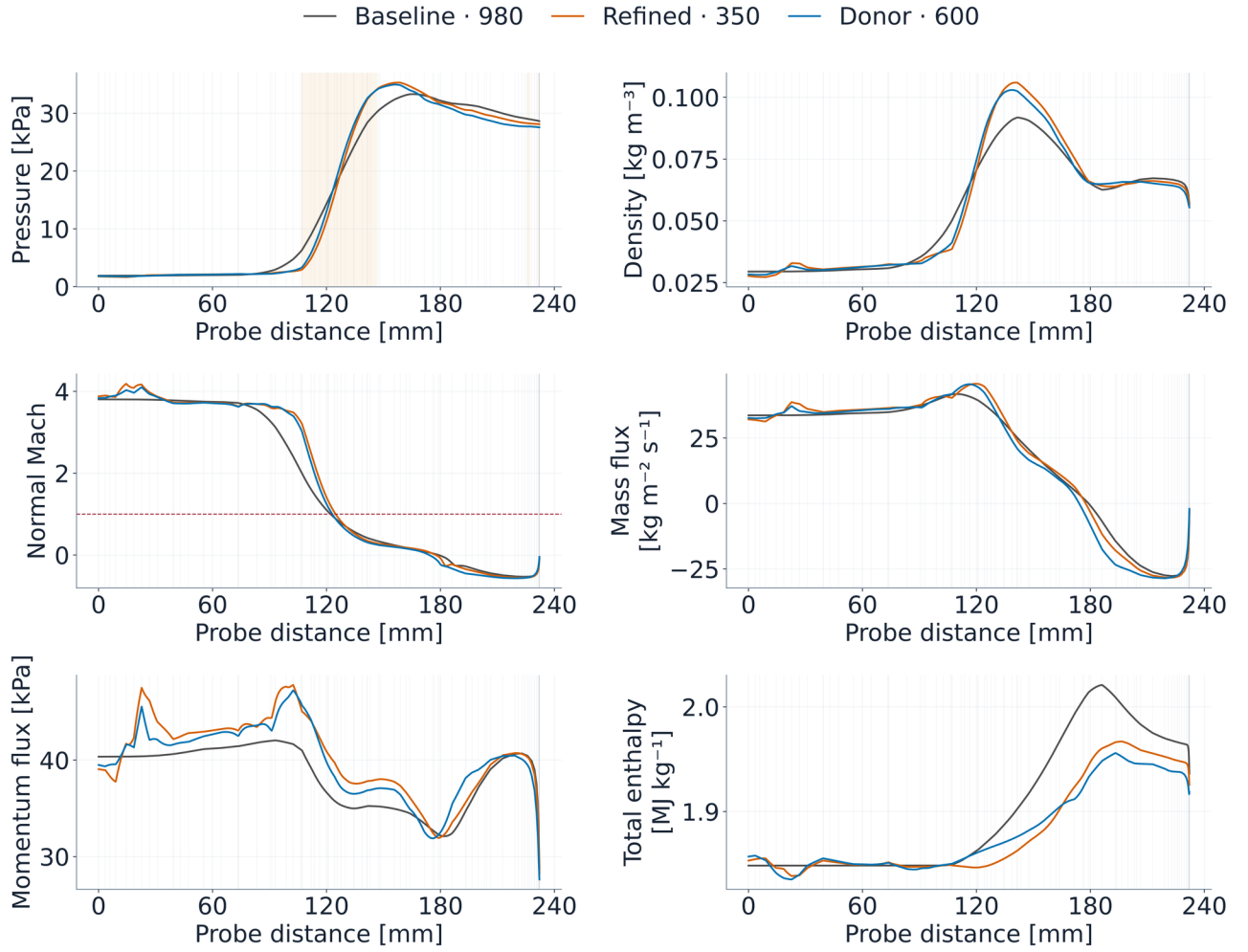


Fig. 51 Historical intake native-3D transect at identical FP64 coordinates. Gray marks sampled cell boundaries; teal denotes compression-supported cores and amber identifies unqualified pressure cores.

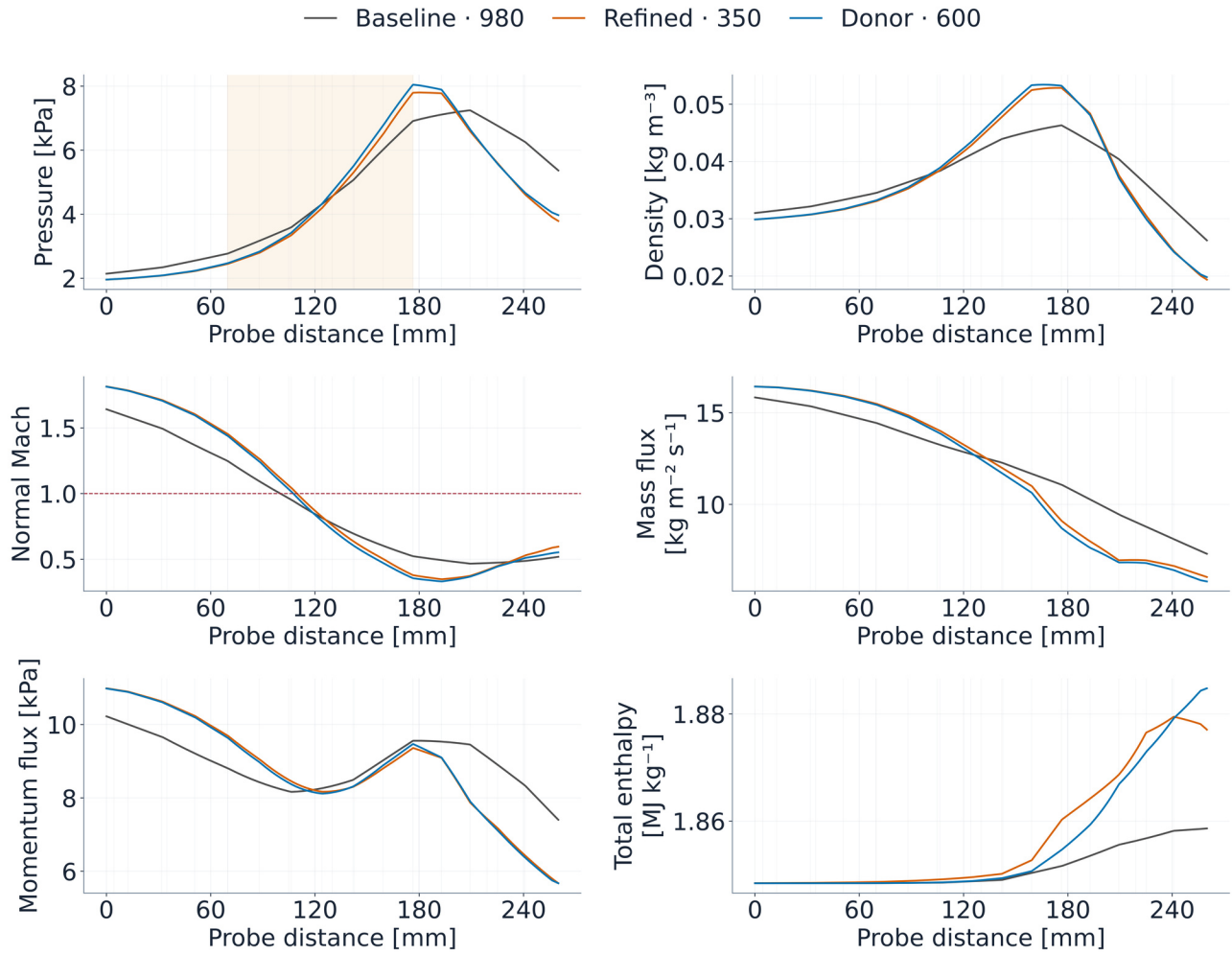


Fig. 52 Historical lower-body native-3D transect at identical FP64 coordinates. Gray marks sampled cell boundaries; teal denotes compression-supported cores and amber identifies unqualified pressure cores.

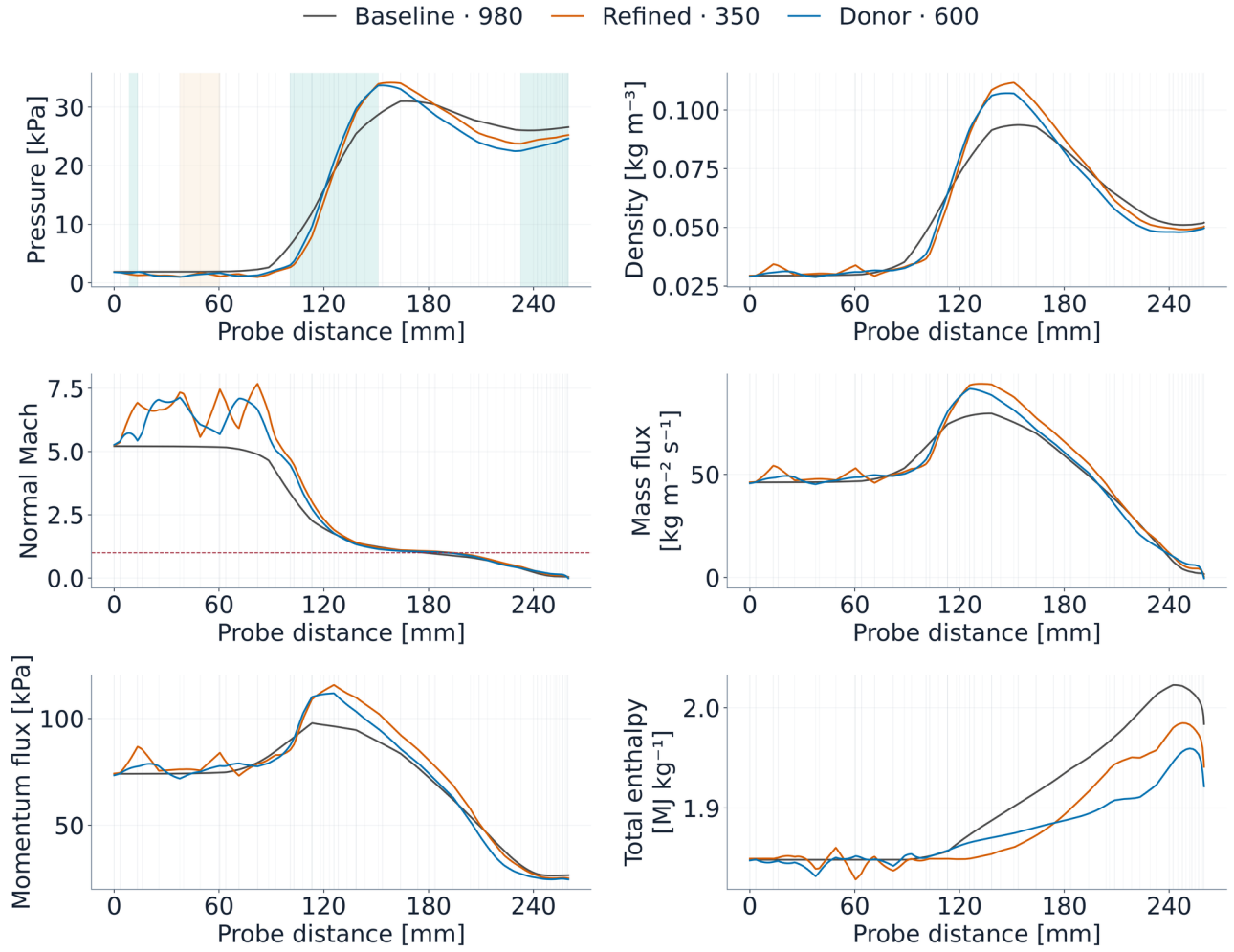


Fig. 53 Historical intake $y = 0$ transect at identical FP64 coordinates. Gray marks sampled cell boundaries; teal denotes compression-supported cores and amber identifies unqualified pressure cores.

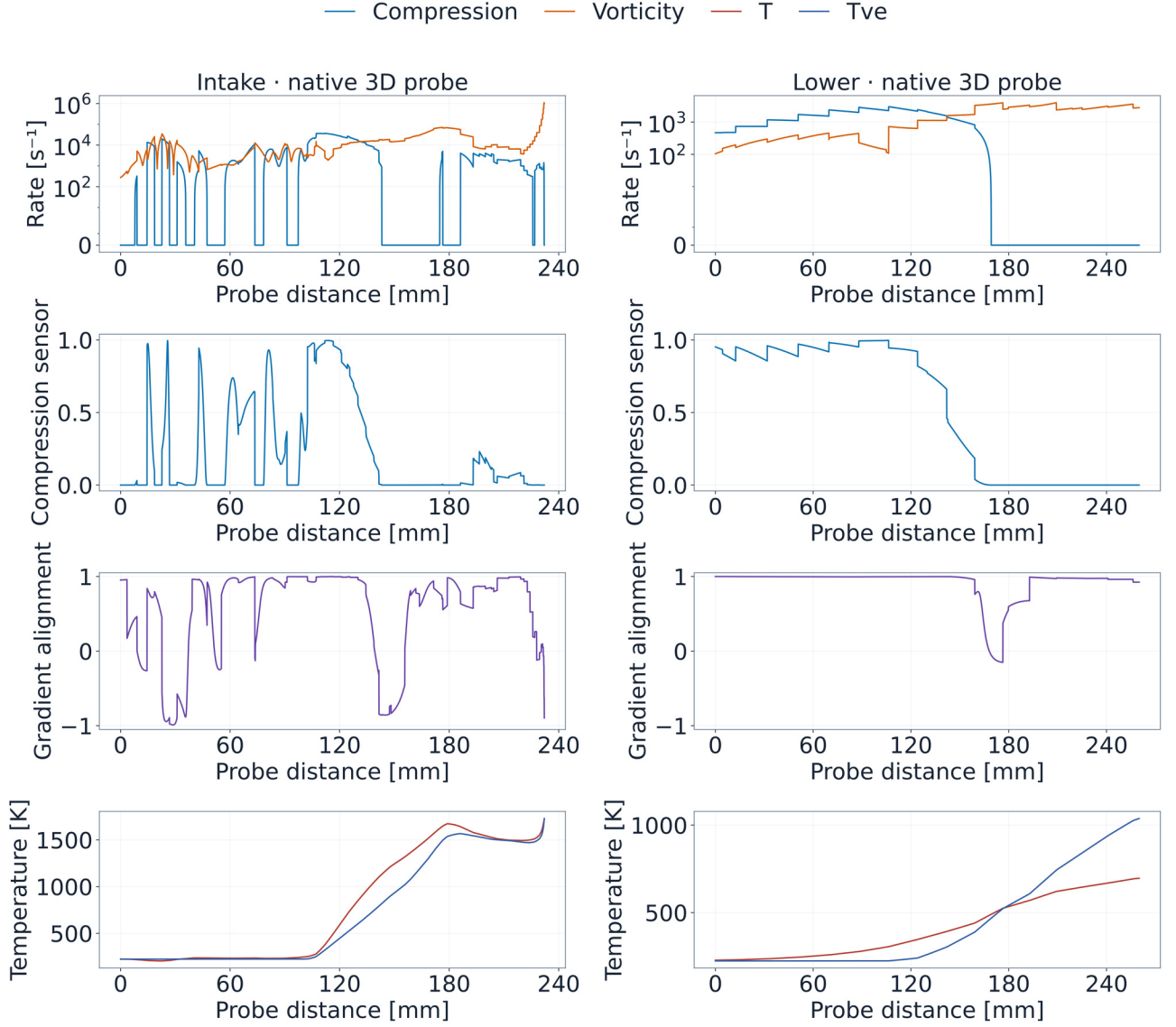


Fig. 54 Checkpoint 600. Native Q1 saved-field derivatives on FP64 geometry. Compression ratio is an indicator; no universal acceptance threshold.

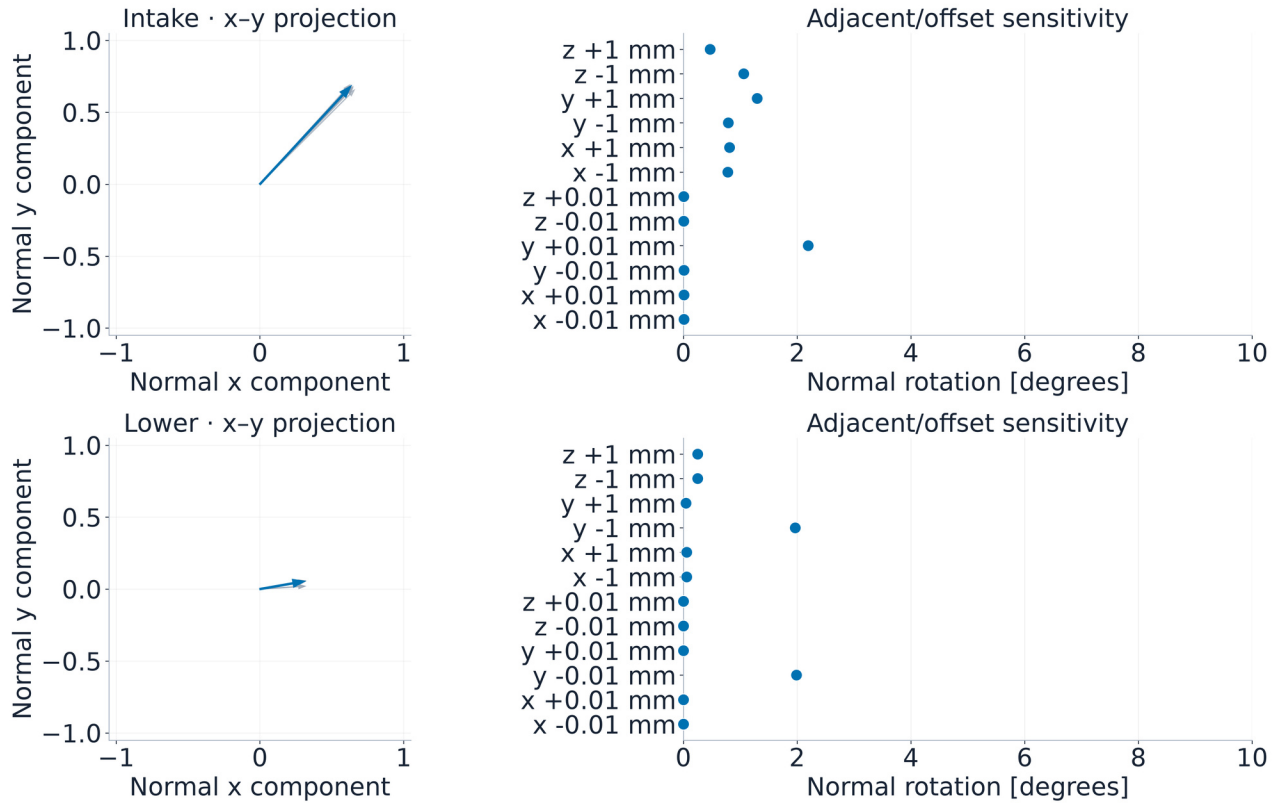


Fig. 55 Native pressure gradients at the anchor and $\pm 0.01 / \pm 1$ mm offsets. Large angular changes identify cell, curvature or interaction sensitivity.

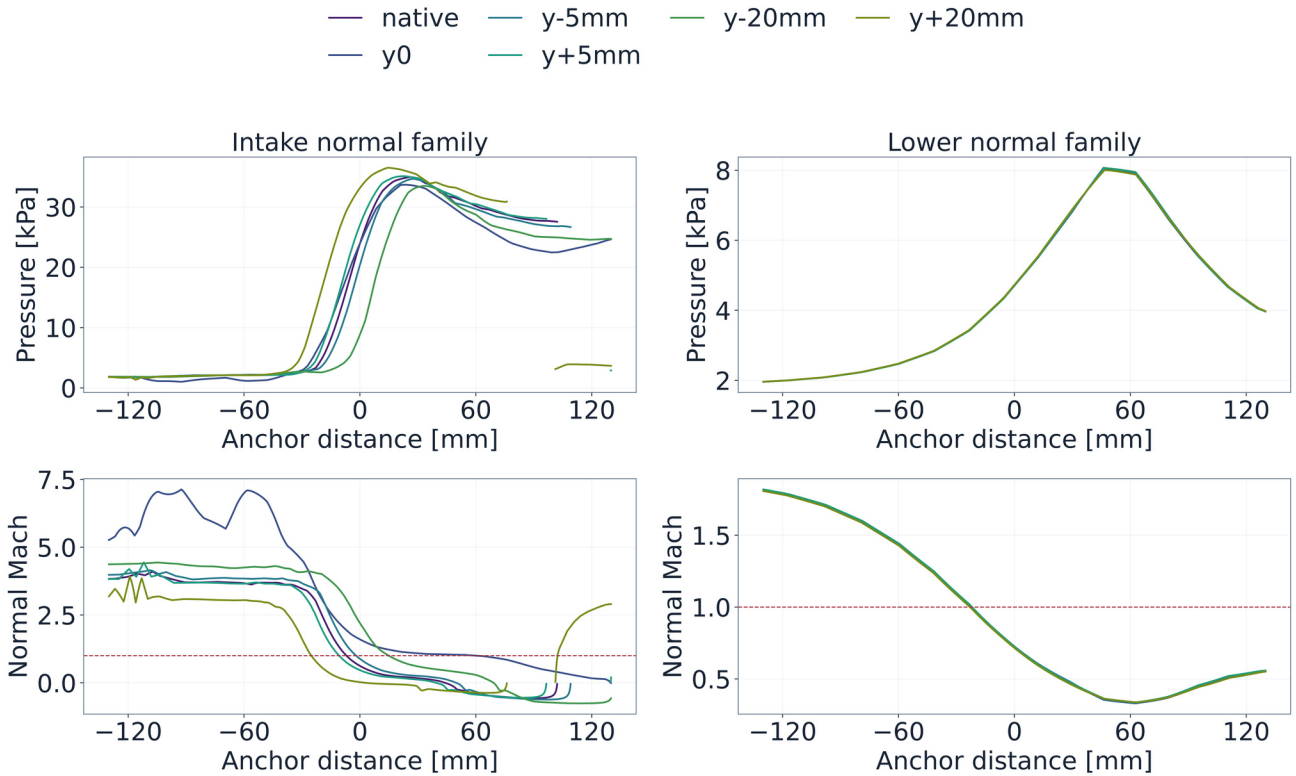


Fig. 56 Checkpoint 600. Each curve uses its declared local anchor and direction; these curves are not the same spatial transect.

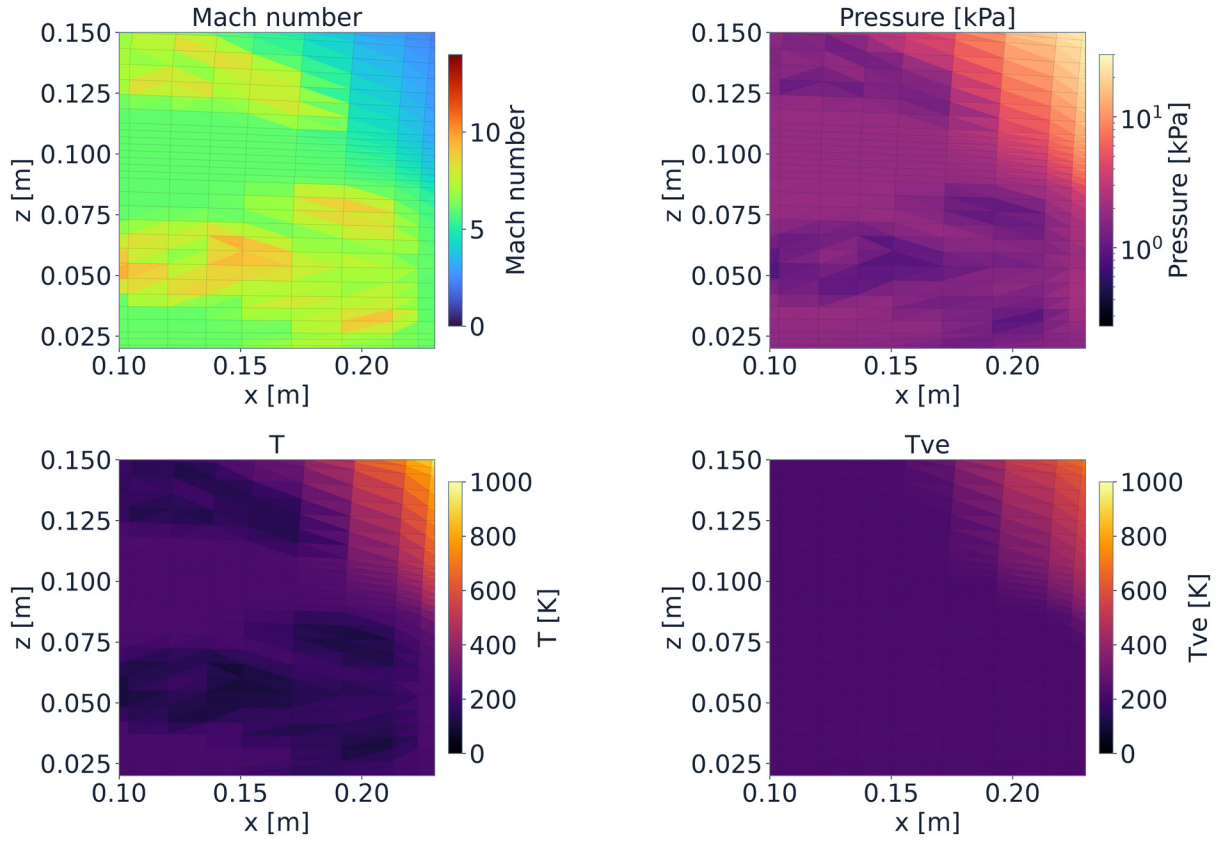


Fig. 57 Checkpoint 600, $y = 0$. Native section edges. Matching-box extrema are recorded for both grids; causation is not inferred from colors.